Three Essays on Hidden Liquidity in Financial Markets

DISSERTATION

zur Erlangung des akademischen Grades
doctor rerum politicarum
(Doktor der Wirtschaftswissenschaft)

eingereicht an der
Wirtschaftswissenschaftlichen Fakultät
der Humboldt-Universität zu Berlin

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Tag des Kolloquiums: Berlin, 19. November 2013

Man muß rechtschaffen sein in geistigen Dingen bis zur Härte, [...] Man muß gleichgültig geworden sein, man muß nie fragen, ob die Wahrheit nützt, ob sie einem Verhängnis wird ... Eine Vorliebe der Stärke für Fragen, zu denen niemand heute den Mut hat; der Mut zum Verbotenen; die Vorherbestimmung zum Labyrinth. Eine Erfahrung aus sieben Einsamkeiten.

Friedrich Nietzsche

Abstract

In recent years, the proliferation of hidden liquidity in financial markets has increased dramatically and shifted to the centre of regulatory debates and market microstructure panels. Yet investors, scientists and policy makers are at odds about its implications and the adequate regulatory responses. Key issues are: 1) What are the main determinants of hidden liquidity? 2) Are hidden orders used by informed or uninformed investors? 3) How does hidden liquidity affect market quality? This thesis addresses these issues in three separate chapters on both empirical and theoretical grounds.

Chapter 1 provides an empirical investigation of the determinants and impact of hidden order submissions. We report that the cross-sectional variation of hidden liquidity is well explained by observable market characteristics, foremost the spread and the tick size. Second, our results suggest that the impact of hidden liquidity is substantial. The submission of large hidden orders has a larger impact on returns than relevant earnings announcement news. Overall, our results suggests that hidden liquidity increases market volatility and trading frictions.

Chapter 2 proposes a structural trading model. We investigate trader's optimal trading strategies with respect to order-exposure in limit order book markets. The optimal exposure size marks a trade-off between costs and benefits of exposure. Our model provides explicit characterisations of the optimal exposure size for various market specifications. Model parameters and exposure strategies are estimated through high-frequency order book data. Our results suggest that hidden orders can significantly increase trade performance.

Chapter 3 develops a dynamic equilibrium model with a public primary market and an off-exchange trading mechanism. Our theory correctly predicts the key findings of the previous chapters. For instance, in line with chapter 1, we show that large hidden orders cause excess returns and increase market volatility; we correctly predict the role of the observable market characteristics in the origination of hidden liquidity. Further, in line with chapter 2, we show that hidden orders can be beneficial under certain market specifications. We verify the theoretical predictions using high-frequency order book data.

Zusammenfassung

An den Handelsbörsen der Welt, hat der Anteil *unsichtbarer Luidität* in den letzten Jahren dramatisch zugenommen. Obwohl dieser Trend zunehmend in den Fokus regulatorischer Debatten und akademischer Dikussionen rückt, sind sich Forscher und die Aufsichtsbehörden über die Implikationen und entsprechende regulatorische Maßnahmen uneins. Zentrale und noch offene Fragestellungen sind: 1) Was sind die Marktfaktoren, die zu einer vermehrten Inanspruchnahme unsichtbarer Order führen? 2) Werden unsichtbare Order von informierten oder uninformierten Investoren benutzt? 3) Welchen Effekt hat ein zunehmender Anteil unsichtbarer Liquidität hinsichtlich der Effizienz der Märkte? In der vorliegenden Arbeit, werden diese Fragestellungen in drei separaten Kapiteln theoretisch und empirisch untersucht.

Mit Hilfe eines speziellen NASDAQ Datensatzes, werden in Kapitel 1 die Marktfaktoren, die unsichtbaren Liquidität begünstigen sowie den Einfluß, den unsichtbare Liquidät auf Märkte ausübt, empirisch ausgewertet. Wir zeigen, daß die Querschnittsvariation unsichtbarer Liquidität entlang des Aktienuniversums in einem hohen Maße durch sichtbare Markteigenschaften erklärt wird, insbesondere durch *spread* und den *tick*. Darüberhinaus zeigt unsere Analyse, daß unsichtbare Order einen starken Einfluß auf Preisreaktionen ausüben. Die beobachteten Preisreaktionen sind zuweilen stärker ausgeprägt als für kursrelevante Meldungen über Unternehmensgewinne. Die empirischen Meßergebnisse geben Grund zu der Annahme, daß Märkte mit hoher unsichtbarer Liquidät volatiler sind und höheren Marktreibungen ausgesetzt sind.

In Kapitel 2 entwickeln wir ein strukturelles Handelsmodell und untersuchen die optimale Entscheidung über sichtbares Handeln. Der Händler muss die Vorteile und Nachteile sichtbaren und unsichtbaren Handelns gegeneinander abwägen. In diesem Rahmen leiten wir für verschiedene Marktspezifikationen explizite Charakterisierungen der optimalen *exposure size* her. Wir schätzen Modelparameter und berechnen die *exposure size* anhand von hoch-frequenten Orderbuchdaten. Unsichtbare Order zeigen unter anderem eine signifikante Verbesserung der Handelsperformance.

In Kapitel 3 entwickeln wir ein dynamisches, spieltheoretisches Handelsmodell mit einer öffentlichen Handelsbörse und einem außerbörslichen Handelsmechanismus. Das Besondere an diesem Model ist, daß es die in den vorhergehenden Kapiteln identifizierten Vor- sowie Nachteile unsichtbarer Liquidität in einem theoretischen Rahmen vereint. Übereinstimmend mit Kapitel 1, sagt das Model voraus, daß große unsichtbare Order signifikante Preisreaktionen hervorrufen. Der Grund liegt in negativen Liquiditäts-Externalitäten: eine stärkere Marktintransparenz schwächt die Koordination zwischen der Angebots- und Nachfrageseite und generiert erhöhte Preisfluktuationen. Da dies nur für große Handelsvolumina der Fall ist, kann der Gebrauch nicht-sichtbarer Order für mittel-große Handelsvolumen durchaus von Vorteil sein. Dies ist wiederum in Einklang mit den Resultaten des zweiten Kapitels. Darüberhinaus, werden in dem Model die in

dem Kapitel 1 beschriebenen Effekte der *tick size* und des *spread* hinsichtlich unsichtbarer Liquidität korrekt vorhergesagt. Weitere zentrale Aspekte der Theorie werden anhand von hoch-frequenten Orderbuchdaten im Rahmen einer *Generelized Impulse Response* Analyse überprüft.

Acknowledgment

There are several people and institutions that played an important role in the completion of this thesis. Their contributions deserve special recognition.

First and foremost, I feel indebted to my supervisors *Prof. Ulrich Horst* and *Prof. Nikolaus Hautsch* for entrusting me with this engaging project. Prof. Horst gave me the chance to join the *Quantitative Products Laboratory* (QPL) and work on a very exciting topic in financial markets research. He has always set high standards with respect to academic work and his demanding approach has immensely enhanced the rigour and structure of my theoretical analysis.

I owe special gratitude to Prof. Hautsch for his sincere interest in this research project and for welcoming me at the *econometrics chair* of the Humboldt-Universität zu Berlin. I and this thesis, in particular, greatly profited from his econometric expertise, sharp focus and professionalism. His constant encouragement and assistance were essential in shaping the mindset of being creative, exploring new ideas and challenging existing scientific conventions.

Several institutions have provided an exceptional academic working environment during the course of my studies. Among them, the QPL granted a fantastic interface between academia and financial industry. The close interaction with practitioners of financial business has enormously enhanced the understanding of market microstructure mechanisms, helped to identify key issues and substantially contributed to the main ideas and intuitions that lay the ground for much of the work in this thesis and certainly for more academic projects to come.

The *econometrics chair*, the *economics department* and the *collaborative research centre 649 Economic Risk* were a constant source of inspiration, insight and widening of scientific horizons. Indeed, I have come a long way from being a tiny myopic limit order book analyst to understanding some of the core principles and main linkages of financial markets theory: microstructure dynamics, pricing kernels and the theory of bubbles and herding, to mention a few.

I want to send my gratitude to *Prof. Georg Weizsäcker* for giving me the privilege and pleasure to co-lecture the seminar *Theory of Market Microstructure* during the summer term 2013. It was an insightful and unique experience.

It was a great pleasure to meet *Bruce Lehmann* (University of California, San Diego). I am grateful for his encouraging words, helpful comments and numerous inspiring discussions on the theory of markets.

This thesis would not have been possible without the support and close cooperation from *Deutsche Bank*. I particularly benefited from the close mentoring and discussions with two exceptionally bright market specialists and senior *quants* of Deutsche Bank: *Mark DiBattista* and *Boris Drovetsky*. I feel extremely indebted to both former leading directors at Deutsche Bank, *Dr. Marcus Overhaus* and *Daniel Nehren* (JP Morgan Chase) for inviting me to New York and work at Deutsche Bank's trading floor. I further want to thank *Andrew Ferraris* (UBS), *Liang Liu* (Bank of America) and *Roel Oomen* (Deutsche Bank) for lots of insightful discussions.

I particularly appreciate the incredible work of the Lobster development team: Jonas Haase, Gustav Haitz and Dr. Ruihong Huang. The Lobster interface has provided high-quality order book data throughout my studies.

It has been a particular pleasure to work alongside my dear colleagues from the *Quantitative Products Laboratory*, *Deutsche Bank*, *Technical Universität Berlin* and *Humboldt-Universität zu Berlin*. The numerous private and academic discussions at lunch time made academic life absolutely enjoyable and intellectually stimulating. My sincere thanks and regards go to Dr. Christoph Baumgarten, Christopher Boortz, Dr. Bernd Droge, Dr. Ruihong Huang, Simon Jurkatis, Dr. Selim Gökay, Maria Grith, Dr. Axel Groß-Klußmann, Jonas Haase, Gustav Haitz, Dr. Robert Huth, Dr. Peter Kratz, Dr. Felix Naujokat, Dr. Peter Malec, Pjotr Mayer, Dr. Andrija Mihoci and Michael Paulsen.

I am specially grateful to my dear friend and former colleague at the department of physics at the Humboldt-Universität zu Berlin, Dr. Fabian Spill (University of Oxford). I owe a heavy portion of *critical thought* and intellectual spirit to our numerous and relentless debates at the department until midnight. On these occasions, we gained invaluable insights outside of the standard curriculum on issues ranging from the Riemann conjecture, to string theory, complex matter systems, the Maxwellian daemon, Quantum chromodynamics, functional analysis, politics and history.

But probably more than to anyone else, I owe the highest gratitude to my lovely family, in particular, my wife Hacer, my mother Fatma, my father Salih and my sister Göksen for their enduring support and care.

Berlin, July 2013
Gökhan Cebiroğlu

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Introduction

Financial markets are increasingly playing a key role in modern economies. Their central task is to facilitate an *efficient transfer of assets and goods* among economic agents. Nowadays, most of the major stock exchanges operate as *limit order* or *order-driven* markets. In these markets, at least one market participant has to submit a limit order, expressing desired trade quantity and prices. This information is stored in the *limit order book* and is readily observable by the open public and other market participants. Those investors who are willing to pose as a *counter-party*, either submit a market order or a limit order at the same price. Hence, in modern trading mechanisms, exposure of trade interests lies at the very basis of trading itself.

Yet over the recent decade, exchanges have drastically increased the proliferation of *hidden orders*, as a growing trading population is interested in trading without disclosing their trade intentions. This raises the concern whether a reduction in pre-trade transparency does actually harm *market quality* and *price discovery*. This is an ongoing and highly controversial debate among researchers, traders and investors and sits high on the agenda of policy-making bodies. The release of the European Commission's impact assessment to the new proposal for a *Directive for Markets in Financial Instruments* (MIFID) echoes the growing regulatory concern:

> "... an increased use of [hidden liquidity] raises regulatory concerns as it may ultimately affect the quality of the price discovery mechanism on the 'lit' markets. [...] The issue at stake is to balance the interest of the wider market with the interest of individuals...".

Ultimately, to identify the right balance, decision makers need a decent assessment of hidden liquidity by taking both perspectives into account: the individual investor's perspective and the wider markets or policy maker's perspective. On an individual level, investors value trading strategies and order types by their ability to reduce transaction costs. There are two ways how hidden orders can contribute to this aim. First, detecting hidden orders and trading against them can provide better prices than visible orders. Second, the trader can use hidden orders themselves and reduce transaction costs by reducing exposure. Since individual trading gains do not necessarily translate into aggregate or *collective* gains, a third issue is whether hidden orders reduce trading frictions and make trading mechanisms more efficient overall. In this thesis, we address the above issues in three separate chapters on both empirical and theoretical grounds. Our analysis benefits from a unique and novel pool of data.

Chapter 1 provides an explorative analysis about hidden liquidity and its main properties. Using unique and novel datasets, we identify the main determinants of traded and submitted hidden order volumes for the S&P 500 in the period between October 2008 and March 2009. We show that the cross-sectional presence of hidden liquidity is well explained by observable and readily available stock characteristics. Using a *Least Angle Regression*, we identify the hierarchy of variables with highest explanatory power. We find that the spread is most significantly related with the presence of hidden liquidity. Moreover, we analyse the inter-temporal properties of hidden liquidity. Hidden orders arrive *sporadically* but in large volumes and they cluster around few price quotes. Finally, we assess the impact of hidden order submissions on several market dimensions. Our results show a striking feature. While the price impact of displayed orders is low, the price impact of large hidden orders exceeds the impact of important *earnings announcement news*. In line with the hypothesis about *liquidity externalities* in chapter 3, our results altogether indicate that hidden orders induce market frictions and price inefficiencies by increasing the likelihood of a mismatch between liquidity supply and demand.

Chapter 2 proposes a structural model to assess the optimal exposure size under limit order book dynamics. The model captures the trade-off between the benefits and costs of exposure. While hiding trade intentions reduces exposure impact, exposing trade intentions can reduce execution risk. Under various market specifications, we derive explicit characterisations of the optimal exposure size. Our framework is particularly amenable to the analysis of high-frequency order message data. We use ITCH data from the INET exchange to estimate model parameters and optimal exposure sizes for a wide range of market specifications. Our results show that exposure impact primarily materialises in tighter liquidity competition. Overall, we find that hidden orders can significantly improve trade performance and are most advantageous when order size is large and same-side liquidity competition is low.

Chapter 3 develops a dynamic equilibrium model between three traders: a hidden trader, a liquidity competitor and a latent block trader. Latent block traders are large traders that have discretion over the trading place. They can either trade in the public primary market (i.e. downstairs market) or in the anonymous off-exchange market (i.e. upstairs market). The hidden trader has discretion about the exposure size. Because large exposed orders have the *critical mass* to elicit demand from the latent block trader, in equilibrium, traders tend to openly display large orders more than medium-sized orders. Hence, exposure triggers positive liquidity externalities and enhances the coordination of liquidity supply and demand. Our framework, allows to derive a range of predictions: 1) Markets with wider spreads and low depth have a higher proportion of hidden liquidity. 2) Large hidden orders beget volatility and increase price inefficiencies. 3) In line with chapter 1, our model correctly predicts that large hidden orders cause excess price returns while displayed orders do not. Finally, employing a Generalised Impulse Response analysis, we succeed in verifying the dynamic implications of our theory.

Chapter 1

Determinants and Impact of Hidden Liquidity: An Empirical Investigation

This chapter is based on Cebiroglu and Horst (2011).

1.1 Introduction

Hidden liquidity has become an indispensable feature of today's electronic exchanges. Numerous statistics show that hidden liquidity comprises a substantial and growing proportion of overall market liquidity. Yet although issues like origination, determinants and impact of hidden liquidity are of pivotal importance to investors, regulators and exchange operators alike, conclusive insights about these issues still remain elusive. Central and actively debated issues are whether hidden liquidity impairs market efficiency and whether hidden orders are used by informed or uninformed traders.

This paper contributes to this debate by providing an explorative and empirical analysis of the main characteristics of hidden liquidity in public exchanges. Specifically, we address three issues. First, we identify the observable market properties that play the most decisive role in the origination of dark liquidity. Second, we look at the spatial and time dispersion of hidden liquidity to infer whether hidden order submission takes place on an individual or collective basis and whether hidden order submissions are *sporadic* events or take place regularly. Third, we assess the ex-post impact of hidden order submissions on different market dimensions.

Literature on Hidden Liquidity

Most of today's order-driven electronic exchanges provide hidden liquidity in the form of specific order types, the most prominent of which is the so-called *Iceberg order*. An Iceberg order is a passive order that has been split into smaller parts of which just a small proportion is visible to the public (if at all). In line with Kyle (1985), empirical studies suggest that these orders are particularly relevant for investors who prefer to reduce *information leakage* (c.f. Foster and Vishwanathan (1990) and Keim and Madhavan (1995)).

A series of empirical studies confirms the growing and substantial use of hidden orders among most major stock exchanges. For instance, Pascual Gasco and Veredas (2008) report that 26% of all trades on the Spanish Stock Exchanges involve hidden volume. Frey and Sandas (2009) find that 9.3% of submitted and 15.9% of executed shares contain Iceberg orders on the German Xetra Stock Exchange. De Winne and D'Hondt (2004, 2007, 2009) report that 27.2% (20.4) of the total liquidity in the book is hidden for the French CAC40 (Belgian BEL20) exchanges and moreover that the hidden ratios can even reach 50% at the best limit prices. Tuttle (2003) finds that around 25% of liquidity for all NASDAQ National Market quotes are hidden. Further studies confirm that hidden liquidity is particularly prevalent among large investors: D'Hondt et al. (2004) report that 81% of orders with total sizes in the largest quartile are Iceberg orders or (partly) hidden orders. Independently, Frey and Sandas (2009) find that Iceberg orders are on average 12-20 times larger than limit orders.[1]

The rationale for hidden trading is rooted in the downsides of information leakage and *exposure risk*. For instance, Copeland and Galai (1983) identify *adverse-selection* risk as the prime motif of hidden order submissions. Harris (2003) attributes exposure risk to the presence of so-called *parasitic traders*. He argues that, at the expense of limit order traders, these *parasitic traders* traders exploit the free trading option of limit orders by *undercutting* them. As a consequence, the limit order trader suffers higher liquidity competition and eventually higher execution risk. Among more theoretical works, Moinas (2010) suggests that *informed* traders *scare-away* liquidity demanders by exposing their intentions. Buti and Rindi (2013) provide a dynamic limit order book model that confirms the intuition provided by Harris: exposure increases liquidity competition at the same side of the market.

Evidently, exposure impact can impact both liquidity supply and liquidity demand. Cebiroglu and Horst (2013) provide a structural model that simultaneously captures exposure impact on both liquidity dimensions in a parametric way. By estimating the structural model parameters, they find that exposure predominantly affects the supply side of liquidity by increasing liquidity competition.

Although these and further empirical findings (c.f. (Bessembinder et al., 2009) and Frey and Sandas (2009)) suggest that hidden liquidity provision can be beneficial to individual investors in certain cases, it is still an ongoing debate whether there is an overall benefit in the supply of hidden liquidity. For instance, while findings in Aitken et al. (2001), Anand and Weaver (2004), Tuttle (2003) and Frey and Sandas (2009) indicate that hidden liquidity provision attracts additional liquidity to the market. Hendershott and Jones (2005) show that overall market quality in the Island electronic communication network (ECN) deteriorated after a regulatory enforcement to stop displaying its limit order book. Edwards et al. (2004) and Bessembinder and Maxwell (2008) also find that market opaqueness can harm market quality.

[1] For more empirical evidence, see Bessembinder et al. (2009), Aitken et al. (2001) and De Winne and D'Hondt (2007).

Our Contribution

Determinants and Cross-sectional Variation of Hidden Liquidity

Using the unique NASDAQ ModelView dataset, we conduct cross-sectional regressions for traded and posted hidden depth for the stocks of the S&P 500 index. We find that there is a significant cross-sectional link between observable stock characteristics and the presence of hidden depth. Observable statistics can explain 70% of the cross-sectional variation of the proportion of hidden depth and 94% of proportion of traded hidden depth.

To investigate the hierarchy among observable explanatory variables, we employ a *forward model selection* procedure, the *Least Angle Regression* (LARS). The average spread is the most powerful stock statistic. It alone accounts for 41% (98%) of explanatory power of a full model specification. Altogether, price characteristics like spread and tick (i.e. inverse of the stock price) capture most of the explanatory power, while liquidity proxies like trading volume, volatility and depth have less explanatory power.

These findings distinguish from earlier studies. For instance, De Winne and D'Hondt (2007), Bessembinder et al. (2009) and more recently Hautsch and Huang (2011) study the inter-temporal predictability of single hidden orders. However, due to data limitations, they do not identify all hidden orders. Instead, our work focuses on the question how hidden depth aggregates in the cross-section of the stock universe. This perspective is more relevant for portfolio managers. In that sense, our approach is closer linked to De Winne and D'Hondt (2009). However, using a unique dataset, we extend their work by using a larger sample size ($N = 448$), incorporating additional stock characteristics like the spread, price, overall depth, inter-trade time interval, trade size and trade volume and analyse both, traded and posted hidden liquidity. Compared to their findings, our results show a strikingly higher statistical significance in terms of r^2 goodness-of-fit.

Hidden Liquidity Clustering

Time-series evolution of hidden depth indicates that hidden liquidity is a sporadic event, i.e. concentrating around few quotes and points in time. To test this observation in a robust way, we define a range of dispersion measures: *entropy*, the *coefficient of variation* and a third *concentration measure*. Our estimates suggests that hidden depth clusters around few price quotes and enters the market sporadically. For instance, we report that while 80% of total *displayed* depth at the ten best price quotes is concentrated on five prices quotes on average, hidden depth concentrates on only 2-3 price quotes. These findings suggest that the presence of hidden depth is associated with single investors.

The Impact of Hidden Liquidity

The evidence that hidden orders are submitted by individual investors leads to a second question: What are the motifs of these single traders and how does the submission of single large orders affect the market? In particular, are hidden trades associated with informed trading? For this purpose, we use an event-study framework to assess the ex-ante and ex-post impact on the different market dimensions.

Our findings are as follows: First, large hidden (orders) are associated with significant ex-post returns, displayed orders are not. The impact of large hidden orders is economically significant as it exceeds the impact of important earnings announcement news. Second, our results suggest that (large) hidden orders are more likely to be submitted by traders who follow a trend, when volatility is low and spreads are narrow. By assessing the impact on the wider market dimension, our analysis complements earlier studies on the impact of hidden orders on execution quality (c.f. Bessembinder et al. (2009)).

However, we do not suggest that hidden orders are associated with informed trading out of two reasons. First, we find that that the ex-post return impact depends on the order size, i.e. the larger the order to be traded, the larger the return impact. Second, the return impact follows qualitatively a *square-root law* in time. This *size* or *quantity* effect is known to be associated with the price impact of market orders (c.f. Farmer and Lillo (2003)). Both observations together provide strong indications that the observed price effects are not due to *information arrival*, but due to the *price impact* or *liquidity effect* of trades.

Altogether, our findings support the view that hidden orders cause price inefficiencies. This is in line with the theory proposed in Cebiroglu et al. (2012) and Admanti and Pfleiderer (1991). Accordingly, hidden orders can not attract counter-parties and therefore more often end up being cancelled and re-submitted in terms of *costly* market orders. The fact that a larger fraction of the order has to be traded via market orders does also materialise in higher excess returns.

Cebiroglu et al. (2012) also predict that unconditional market volatility is larger in markets that trade more hidden. We find that when volatility is conditioned on the arrival of hidden orders, there is no significant increase in ex-post volatility. This suggests that the source of randomness lies in the arrival process but that the magnitude of fluctuations is governed by the deterministic market impact of hidden orders.

Outline

The remainder of this paper is structured as follows. Section 1.2 describes the dataset and presents descriptive statistics on traded and posted hidden liquidity. Section 1.3 reports analysis on the determinants of hidden liquidity and its cross-sectional variation. We employ LARS to identify the main determinants of hidden liquidity supply. In section 1.4, we employ an event-study framework to analyse the impact of hidden orders on various market dimensions. Section 1.5 concludes.

1.2 Data

Sample Selection

Our data derives from the NASDAQ ModelView dataset, which contains minute-by-minute snapshots of the full aggregated order book depth, including visible and hidden depth for all S&P 500 stocks that were consistently traded during the time period of October 2008 to March 2009. The order book data is presented in aggregated form, that is displayed and hidden volumes are aggregated to their total depths. To reduce the impact of *outliers*, we restrict our analysis to stocks that show an average daily traded volume (ADV) of less then 50 million shares, 0.2 million average number of trades, average spread of less than 25 cents and average price of less then 100$.[2] We finally obtain a sample size of $N = 448$ shares. To reduce the impact of opening and closing auctions, we constrain our analysis to daily periods between 09:15 and 15:45. Thus, the daily sample size for each stock counts 390 minute-by-minute snapshots. We also consider depth up to the best ten price levels and not beyond.

ModelView provides information about (displayed and hidden) depth only. Data about traded hidden orders is obtained from the Trade-and-Quote (TAQ) data source. TAQ does not discriminate between hidden and displayed trades. We proxy the size of hidden trades by the amount of trades that executed within the best quotes (i.e. within the spread). These trades must have been hidden trades, as hidden orders can not execute at or beyond the best quotes since displayed orders always have priority. We point out that this assumption crucially depends on the reporting mechanism and the latency of reporting.[3]

Besides extracting the unobservable, hidden volumes from the two datasets, we further consider the stock's main observable characteristic statistics: The average daily traded volume (ADV), the average inter-trade time interval ($Time$), average volatility ($HiLo$), average trade size ($TrSize$), average spread ($Spread$), average price ($Price$), and average top of the book depth (Top).

Summary Statistics

Based on the ADV we sort stocks into liquidity quintiles q_1, q_2, q_3, q_4, q_5, starting with the least liquid quintile q_1 and q_5 representing the most liquid one. Table 1.6 in the appendix reports cross-sectional sample statistics on posted and displayed liquidity as taken from the NASDAQ ModelView data set. Table 1.5 reports cross-sectional averages of ADV , $Time$, $HiLo$, $TrSize$, $Spread$, $Price$ and Top and the corresponding posted and traded hidden liquidity volumes by ratio and total volume. $Ratio$ refers to the hidden proportion of total liquidity. Finally, 1.4 reports cross-correlations between the observable stock characteristics.

[2]Trade volumes are extracted from the Trade and Quote Database (TAQ) dataset provided by Deutsche Bank

[3]Assuming that latency is far lower than the average inter-trade-arrival time and that best bid and ask prices are adjusted after each trade immediately, it is a reasonable first approximation to assume that hidden order can only be executed within the spread.

The main findings can be summed up as follows. First, the proportion of supplied hidden depth and traded hidden shares is significant. Overall, we report 17% of supplied depth and 16% of traded depth to be hidden. Second, the hidden liquidity proportion is larger for less liquid stocks. For instance, the proportion of hidden trades reaches 26% for the least liquid quintile and is only about 7% for the most liquid quintile.[4]

1.3 Hidden Liquidity Determinants

1.3.1 Do Observable Variables Explain Hidden Liquidity?

In this section, we examine how several observable stock characteristics (i.e. ADV, $Time$, $HiLo$, $TrSize$, $Spread$, $Price$, and Top) relate to hidden liquidity, in particular the average posted and traded hidden volumes and ratios H_{ps}, H_{tr}, H^R_{ps} and H^R_{tr}. For this purpose, we propose simple linear model as follows:

$$\begin{aligned}
H_{ps} &= \alpha + \alpha_A ADV + \alpha_{Ti} Time + \alpha_V HiLo + \alpha_{Tr} TrSize \\
&\qquad + \alpha_S Spread + \alpha_P Price + \alpha_T Top + \epsilon_1, && (1.3.1)\\
H^R_{ps} &= \alpha^R + \alpha^R_A ADV + \alpha^R_{Ti} Time + \alpha^R_V HiLo + \alpha^R_{Tr} TrSize \\
&\qquad + \alpha^R_S Spread + \alpha^R_P Price + \alpha^R_{To} Top + \epsilon_2, && (1.3.2)\\
H_{tr} &= \beta + \beta_A ADV + \beta_{Ti} Time + \beta_V HiLo + \beta_{Tr} TrSize \\
&\qquad + \beta_S Spread + \beta_P Price + \beta_{To} Top + \epsilon_3, && (1.3.3)\\
H^R_{tr} &= \beta^R + \beta^R_A ADV + \beta^R_{Ti} Time + \beta^R_V HiLo + \beta^R_{Tr} TrSize \\
&\qquad + \beta^R_S Spread + \beta^R_P Price + \beta^R_{To} Top + \epsilon_4. && (1.3.4)
\end{aligned}$$

As quantities like ratio, depth and spread are positive, we also propose a linear model with (partly) log-transformed variables, i.e.

$$\begin{aligned}
\log(H_{ps}) &= \hat\alpha + \hat\alpha_A ADV + \hat\alpha_{Ti} Time + \hat\alpha_V HiLo + \hat\alpha_{Tr} TrSize \\
&\qquad + \hat\alpha_S Spread + \hat\alpha_P Price + \hat\alpha_T Top + \hat\epsilon_1, && (1.3.5)\\
\log(H^R_{ps}) &= \hat\alpha^R + \hat\alpha^R_A ADV + \hat\alpha^R_{Ti} Time + \hat\alpha^R_V HiLo + \hat\alpha^R_{Tr} TrSize \\
&\qquad + \hat\alpha^R_S \log(Spread) + \hat\alpha^R_P Price + \hat\alpha^R_{To} Top + \hat\epsilon_2, && (1.3.6)\\
\log(H_{tr}) &= \hat\beta + \hat\beta_A ADV + \hat\beta_{Ti} Time + \hat\beta_V HiLo + \hat\beta_{Tr} TrSize \\
&\qquad + \hat\beta_S \log(Spread) + \hat\beta_P Price + \hat\beta_{To} Top + \hat\epsilon_3, && (1.3.7)\\
H^R_{tr} &= \hat\beta^R + \hat\beta^R_A ADV + \hat\beta^R_{Ti} Time + \hat\beta^R_V HiLo + \hat\beta^R_{Tr} TrSize \\
&\qquad + \hat\beta^R_S \log(Spread) + \hat\beta^R_P Price + \hat\beta^R_{To} Top + \hat\epsilon_4. && (1.3.8)
\end{aligned}$$

We apply standard assumptions for the models (1.3.1) to (1.3.8), i.e. the stochastic error terms

[4] On an individual basis, some illiquid stocks trade more than 50% of their volume by hidden shares.

$(\epsilon_i)_{i=1,2,3,4}$ and $(\hat{\epsilon}_i)_{i=1,2,3,4}$ are iid and normal. Estimates are obtained by applying the method of *Ordinary Least Squares* (OLS) for each model separately.

Estimation Results

Estimation results are shown in table 1.7. The coefficient estimates for posted liquidity volume H_{ps} and ratio H_{ps}^R are shown in column 1 to 4 for the transformed and non-transformed models. The coefficient estimates for traded hidden liquidity volume H_{tr} and ratio H_{tr}^R are shown in column 5 to 8 for the transformed and non-transformed models.

Our results can be summarised in three points. First, our models show significant *goodness-of-fit*. We report r^2-values in a range starting from 45% to up to 94%, exceeding by far the explanatory power of the models proposed in De Winne and D'Hondt (2007). Likewise, f-statistics show significance beyond the 0.01% level.

Second, observable market characteristics have substantially higher explanatory power after applying suitable transformations and considering *hidden liquidity ratios* instead of *hidden liquidity volumes*. For instance, r^2 of posted hidden liquidity increases from 50% to 70% after transforming and taking ratios. Similarly, the explanatory power increases for traded hidden liquidity from 66% to 94%. T- and f-statistics also increase significantly after applying transformations.

Third, our findings suggest that the average spread appears to be the main indicator for hidden liquidity supply. On a less significant level, hidden liquidity supply is also associated with the average price (i.e. inverse tick size) and the average inter-trade time. The high-low variation does not significantly affect hidden liquidity.

Overall stocks trade on average more hidden when - ceteris paribus - their average spreads are large, their price is high and they trade less volumes. Altogether, these results suggests that hidden orders are used more in less liquid stocks.

1.3.2 Identifying the key Variables

Our findings indicate that there is a hierarchy among the observable variables. For instance, plain hidden volumes appear to be better explained by liquidity quantities while normalised volumes, i.e. hidden ratios, are well captured by price quantities, like spread and midpoint. In this section, we make use of the *Least Angle Regression* (LARS) to capture this hierarchy. Our aim is to identify the most informative predictors.

Methodology: Least Angle Regression (LARS)

Typically, model selection algorithms such as *Lars*, *Lasso*, *All Subsets*, *Forward* and *Backward Elimination* are used and designed to reduce the number of covariates and to identify an efficient and parsimonious set of predictor variables.[5] Efron et al. (2004) explains how the LARS procedure works,

> " [The Lasso and Forward Stagewise Regression] are variants of a basic procedure called Least Angle Regression. [...] [LARS] can be viewed as moderately greedy forward stepwise procedure whose forward progress is determined by compromising among the currently most correlated covariates. LARS moves along the most obvious compromise direction, the *equiangular* vector, while the Lasso and Stagewise procedure put some restrictions on the equiangular strategy".

We briefly sketch the LARS methodology as explained in Efron et al. (2004) using their notation. Lars regression consists of multiple steps. At each step the algorithm builds up successive models and estimates $\widehat{\mu} = X\widehat{\beta}$, so that after k steps the model comprises only k parameters that are nonzero. More precisely, assume we have m linearly independent covariates $x_1, x_2, ..., x_m$. And denote by A some subset of the index set $\{1, 2, ..., m\}$ with cardinality $|\mathcal{A}| = a$ and $\mathbf{1}_{\mathcal{A}}$ the vector of all ones with length equaling a. Efron et al. define the following matrices

$$X_{\mathcal{A}} = (\cdots s_j X_j \cdots)_{j\in\mathcal{A}}, \qquad \mathcal{G}_{\mathcal{A}} = X_{\mathcal{A}}{}' X_{\mathcal{A}}, \tag{1.3.9}$$

$$A_{\mathcal{A}} = (\mathbf{1}_{\mathcal{A}}{}' \mathcal{G}^{-1} \mathbf{1}_{\mathcal{A}})^{-\frac{1}{2}}, \qquad w_{\mathcal{A}} = A_{\mathcal{A}} \mathcal{G}_{\mathcal{A}}^{-1} \mathbf{1}_{\mathcal{A}}, \tag{1.3.10}$$

where $s_j = \pm 1$ and $w_{\mathcal{A}}$ is the unit vector making *equal angles*, less than 90 degrees, with the columns of $X_{\mathcal{A}}$. The so called *equiangular vector* $\mathbf{u}_{\mathcal{A}}$ reads

$$\mathbf{u}_{\mathcal{A}} = X_{\mathcal{A}} w_{\mathcal{A}} \qquad \text{with} \qquad ||\mathbf{u}_{\mathcal{A}}||^2 = 1. \tag{1.3.11}$$

The LARS algorithm can be described as follows. Starting with the estimate $\widehat{\mu}_0 = 0$ one successively builds up $\widehat{\mu}$ in steps. Therefore, assume $\widehat{\mu}_{\mathcal{A}}$ to be the current LARS estimate. Then the current correlation reads

$$\widehat{\mathbf{c}} = X' (y - \widehat{\mu}_A), \tag{1.3.12}$$

where we define the *active* set $\mathcal{A}$ to be the set of indices corresponding to the covariates with the greatest absolute current correlations, i.e.

$$\widehat{C} = \max_j \{|\widehat{c}_j|\} \qquad \text{and} \qquad \mathcal{A} = \{j : |\widehat{c}_j| = \widehat{C}\}. \tag{1.3.13}$$

We define $s_j := sign\{\widehat{c}_j\}$ with $j \in \mathcal{A}$ and we again compute $X_{\mathcal{A}}, A_{\mathcal{A}}$ and $\mathbf{u}_{\mathcal{A}}$ as in (1.3.9)-(1.3.11). Finally, the updated estimate $\widehat{\mu}_{\mathcal{A}_+}$ reads

$$\widehat{\mu}_{\mathcal{A}_+} = \widehat{\mu}_{\mathcal{A}} + \widehat{\gamma}\, \mathbf{u}_{\mathcal{A}}, \qquad \text{with} \qquad \widehat{\gamma} = \min_{j\in\mathcal{A}^c}{}^{+} \left\{ \frac{\widehat{C} - \widehat{c}_j}{A_{\mathcal{A}} - a_j}, \frac{\widehat{C}\widehat{c}_j}{A_{\mathcal{A}} + a_j} \right\}, \tag{1.3.14}$$

[5]This is particularly useful in high-dimensional statistics as simpler models enhance the scientific insights for models with high degrees of freedom. See Gelper and Croux (2008) for an application in time series forecasting.

where $\min^+$ indicates that the minimum is taken only over positive components for each choice of j in (1.3.14). One can easily show that the maximum absolute correlation declines with each step. In other words, the followings holds:

$$\widehat{C}_+ = \widehat{C} - \widehat{\gamma}\, A_{\mathcal{A}}\,. \tag{1.3.15}$$

The Akaike Information Criterion

The LARS procedure provides k model estimates for $\widehat{\mu}$ in k steps. However, one wants to know and choose only the *best* of these models, i.e. the model that finds the right balance between *goodness-of-fit* and parsimony. Let $\mathbf{y}$ denote some dependent variable we want to explain and

$$\mathbf{y} \,\propto (\mu\,, \sigma^2 \mathbf{I}), \tag{1.3.16}$$

indicating that the y_i are uncorrelated, with mean μ_i and variance σ^2. Then we can write

$$(\widehat{\mu}_i - \mu_i)^2 = (y_i - \widehat{\mu}_i)^2 - (y_i - \mu_i)^2 + 2(\widehat{\mu}_i - \mu_i)(y_i - \mu_i). \tag{1.3.17}$$

Summing over i and taking expectation yields

$$E\left[\frac{||\widehat{\mu}\, - \mu\,||^2}{\sigma^2}\right] = E\left[\frac{||\mathbf{y}\, - \widehat{\mu}\,||^2}{\sigma^2} - n\right] + 2\underbrace{\sum_{i=1}^{n}\frac{cov(\widehat{\mu}_i, y_i)}{\sigma^2}}_{=:df}, \tag{1.3.18}$$

where the term *df* is identified to be the models *degree of freedom*, i.e.

$$df := \sum_{i=1}^{n}\frac{cov(\widehat{\mu}_i, y_i)}{\sigma^2}. \tag{1.3.19}$$

The *Akaike information criterion* is defined as

$$C_p := \frac{||\mathbf{y} - \widehat{\mu}\,||^2}{\sigma^2} - n + 2df. \tag{1.3.20}$$

When σ^2 and df are known, C_p is an unbiased estimator of the *true* risk $E\left[\frac{||\widehat{\mu}-\mu||^2}{\sigma^2}\right]$. In order to select the *best* model among a set of models, one chooses the one with the lowest C_p value. This model is associated with the *best* trade-off between bias (*accuracy*) and variance (*complexity*).

Estimation Results

Table 1.8 and 1.9 report results of the LARS procedure for posted and traded hidden liquidity with respect to the models (1.3.1)-(1.3.8). For each LARS-step, we report the selected variable (*action*), its resulting r^2-goodness-of-fit (r^2_{lars}) and its ratio with the adjusted-r^2 of the respective full-models (r^2_{ols}). We also report estimates for C_p.

The results are as follows. Generally, hidden volumes (posted and traded) are mainly affected by liquidity quantities like ADV, $TrSize$, $Time$ and Top than non-liquidity quantities like $Spread$ and $Price$. The picture reverses when considering normalised hidden quantities (i.e. ratios). In this case, non-liquidity quantities like $Spread$ and $Price$ carry most explanatory power. In these cases, the spread ranks first and captures most of the predictive power (see for instance column 4 in table 1.9).

There are some deviations to this general observation, in particular with respect to $TrSize$ and $Price$. For instance, $TrSize$ ranks least for traded hidden volume, although it is a liquidity quantity. However, since $TrSize$ is highly correlated with ADV and $Time$ according to table 1.4, the addition of ADV or $Time$ in early stages of the LARS procedure already incorporates a substantial amount of predictive power of $TrSize$ into the model. Hence, its residual explanatory power may not be sufficient to be selected at later LARS steps. We thus suggest that this effect derives from correlation among predictor variables and indirect or latent causation (i.e. *spurious regression*).

Similarly, we observe that $Price$, although a non-liquidity quantity, ranks highest for hidden traded volume. However, its associated r^2 value reveals that its explanatory contribution is weak. It is a known and general problem that model selection procedures can occasionally select inferior variables. We refer to Weisberg (2004) for a more detailed discussion.

1.4 Hidden Liquidity Impact

In this section, we address the temporal aspects and implications of hidden order submissions. We concentrate on the question 1) whether hidden order submission is localised at a few points in time and prices and hence originate from single submissions. And 2) we discuss whether the presence of hidden liquidity is associated with informed trading or market frictions.

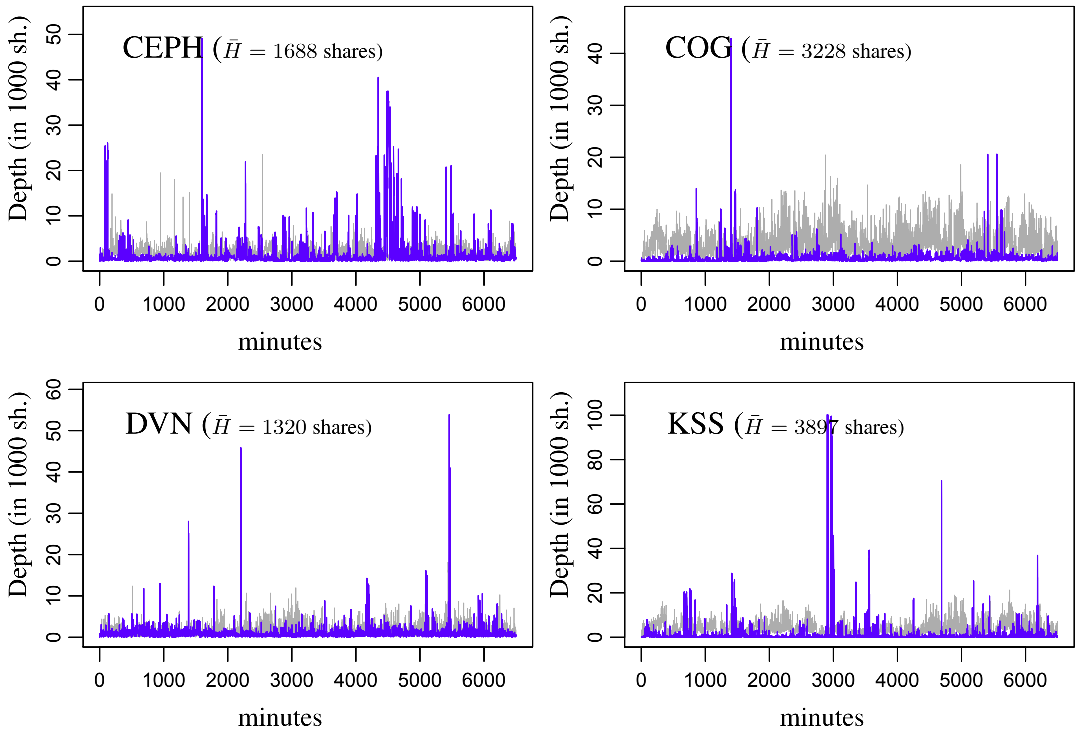
To wit, we recycle an earlier result from table 1.6. From the standard deviation and mean for hidden, displayed and total depth, we construct the *coefficient-of-variation*, $\frac{\sigma}{\mu}$, where σ denotes the standard-deviation and μ the sample mean. Results are shown in table 1.1.

Table 1.1: Coefficients of variation for hidden and displayed liquidity.

	Hidden Depth	Visible Depth	Total Depth
$\frac{\sigma}{\mu}$	2.62	0.55	0.63

The coefficient-of-variation of hidden liquidity exceeds the one for displayed liquidity by a factor of roughly 5. High variation for positive variables suggests that most of the stochastic activity lies at large numbers accompanied with large sequences of no activity at all (i.e. zeros). To backup this claim, we provide a time evolution plot for hidden and displayed depth for four random stocks in figure 1.1. Observe that the hidden depth evolution (blue curve) exhibits *spiky* dynamics, i.e. hidden depth concentrates around single large orders in time, accompanied by extended periods of low-depth activity. On the other hand, displayed depth shows a more regular pattern. The hidden spikes can be significant. For instance, in case of the stock KSS, the hidden

Figure 1.1: Examples for time series evolution of hidden and displayed depth in the order book for 6500 consecutive minutes for a selected stock of the S&P 500. $\bar{H}$ denotes unconditional average of total hidden depth supply.



spike at around the 3000th minute accounts for more than 25 times the average hidden depth.

The very fact that these arguably large liquidity spikes emerge and vanish in the course of only a few minutes provides strong suspicion that they originate from individual and large investors and not a crowd of investors.[6]

1.4.1 Measuring the Concentration of Hidden Liquidity

In this section, we put the above observations to a robust test. To this end, we introduce a range of measures to estimate the degree of *dispersion* or *localisation* and assess the difference between hidden and displayed liquidity. We analyse dispersion along both dimensions, price and time.

[6]This reasoning would be in line with prior empirical findings that suggest that hidden orders are mainly used by large investors, see Bessembinder et al. (2009) for instance.

We introduce some notation. Denote x_{ij}^h (x_{ij}^d) the hidden (displayed) depth at time t_i ($i = 1, ..., n$) at price quote p_j ($0 \leq j \leq m$) for some stock. The hidden and displayed liquidity distribution at time t_i is then given by $x_i^h = (x_{i1}^h, x_{i2}^h, x_{i3}^h, ..., x_{im}^h)$ and $x_i^h = (x_{i1}^h, x_{i2}^h, x_{i3}^h, ..., x_{im}^h)$ respectively. We assume that the list of prices is finite and m denotes the maximum number of price quotes. Moreover, we denote by y_i^h (y_i^d) the total hidden (displayed) depth at time t_i, i.e. $y_i^h = \sum_{j=1}^m x_{ij}^h$ ($y_i^d = \sum_{j=1}^m x_{ij}^h$). Their time evolution is denoted by y^h (y^d), that is to say $y^h = (y_1^h, y_2^h, y_3^h, ..., y_m^h)$ and $y^d = (y_1^d, y_2^d, y_3^d, ..., y_m^d)$ hold.

Concentration and Dispersion Measures

The first measure will be the coefficient-of-variation (C) as in table 1.1. In line with our desire to capture dispersion along both, the time and the price domain, we define two measures accordingly

$$C^{time} := \frac{\sigma(y^q)}{\mu(y^q)}, \qquad C^{book}(i) := \frac{\sigma(x_i^q)}{\mu(x_i^q)}, \tag{1.4.1}$$

where σ denotes the respective sample's standard-deviation and μ its the standard mean.

The second measure is motivated by the concept of entropy.[7] The *temporal* ϕ_{time} and the *spatial* or *order-book entropy* ϕ_{book} read

$$\phi^{time} := -\frac{1}{\log(n)} \sum_{i=1}^{n} g_i^q \log(g_i^q), \tag{1.4.2}$$

$$\phi^{book}(i) := -\frac{1}{\log(m)} \sum_{j=1}^{m} h_{ij}^q \log(h_{ij}^q), \tag{1.4.3}$$

g_i^q and h_{ij}^q denote the corresponding empirical density distributions, i.e. $g_i^q := y_i^q / (\sum_{i=1}^m y_i^q)$ and $h_{ij}^q := x_{ij}^q / (\sum_{j=1}^m x_{ij}^q)$ with $q = d, h$ and $i = 1, ..., n$. The choice of the normalisation factors, $1/\log(n)$ and $1/\log(m)$, where n and m denote the respective sample (state) sizes, ensures that entropy is normalised and values range between 0 and 1. This eases cross-sectional comparison and comparisons across different sample sizes.[8] It is well-known that the entropy measure is non-negative and that it takes on its maximum value for equi-distributed weights (i.e. state of highest dispersion) and its minimum when all but one weight is non-zero (i.e. state of highest of localisation).

Although both measures capture sample dispersion, the obtained numbers hardly allow for an illustrative understanding, particularly in the case of entropy. To account for this deficit, we

[7] In thermodynamics, entropy is understood to represent the degree of dispersion (or disorder) in the thermodynamical system's micro state-space. To be more precise, according to the famous Gibbs formula the Entropy S is defined according to $S = -k_B \sum_i p_i \log p_i$, where p_i represents the probability of a finite system to reside in the state i, where k_B denotes the Boltzmann-constant. Nowadays, entropy finds more and more use in social and economic sciences. For instance, see Hart (1971)

[8] Observe that without the chosen normalisation, entropy would increase with the sample size and thus the notion of *concentration* would consequently depend on the sample size n.

provide an additional measure: *the minimum fraction of elements of a vector that account for at least a fraction s of the total sum of the vector*. More formally, let $\widetilde{x}_i^q$ and $\widetilde{y}^q$ denote the vectors y^q and x_i^q in descending order. We consider their partial sums $\overline{y}_j^q = \sum_{i=1}^{j} y_{l_i}^q$, $j = 1, ..., m$ and $\overline{x}_{ij}^q = \sum_{r=1}^{j} x_{ik_r}$. Then we can define the measures of *concentration* for both the price and the time domain according to

$$L_s^{time} = \frac{1}{n} \underset{\substack{1 \le j \le n \\ \overline{y}_j^q \ge s \cdot \overline{y}_n^q}}{\arg\min} \, \overline{y}_j^q, \qquad L_s^{book}(i) = \frac{1}{m} \underset{\substack{1 \le j \le m \\ \overline{x}_{ij}^q \ge s \cdot \overline{x}_m^q}}{\arg\min} \, \overline{x}_{ij}^q. \tag{1.4.4}$$

The smaller this number, the fewer states occupy s percent of the overall depth. To check that this construction does well behave, consider the case $y^q = (0, 0, 0, 0, Q)$ with $Q > 0$. We have $\overline{y}_j = Q$ for all $j \ge 1$. Hence,

$$L_s^{time} = \frac{1}{5} \underbrace{\underset{\substack{1 \le j \le 5 \\ \overline{y}_j^q \ge s \cdot Q}}{\arg\min} \, \overline{y}_j^q}_{=1} = \frac{1}{5} \cdot 1 = \frac{1}{5} = 0.20 \tag{1.4.5}$$

Indeed 20 percent of the state $j = 5$ occupy more than the fraction s of the total depth.

Cross-sectional averages for the time dispersion measures are obtained in the usual way. For instance, consider the estimates for the time entropy of stock k, i.e. ϕ_k^{time} as per (1.4.3). We construct the cross-sectional average as follows

$$\phi^{time} := \frac{\sum_{k=1}^{n} \sigma^{-2}(\phi_k^{time}) \phi_k^{time}}{\sum_{k=1}^{n} \sigma^{-2}(\phi_k^{time})}. \tag{1.4.6}$$

Similarly, to obtain cross-sectional averages for the price dispersion measures, we first construct the measures for each time and stock and average first over time and afterwards over the cross-section. For instance, consider the price entropies for each stock and time, i.e. $\phi_k^{book} = \frac{1}{n} \sum_{i=1}^{n} \phi_k^{book}(i)$. And denote $\sigma^2(\phi_k^{book})$ the respective standard variance. Then we define the cross-sectional average according to

$$\phi^{book} := \frac{\sum_{k=1}^{n} \sigma^{-2}(\phi_k^{book}) \phi_k^{book}}{\sum_{k=1}^{n} \sigma^{-2}(\phi_k^{book})}. \tag{1.4.7}$$

The same procedure is applied to the other measures, i.e. C^{time}, C^{book} and L^{time}m L^{book}.

Estimation Results

Results are shown in the tables 1.2 and 1.3 and grouped into liquidity quintiles. Estimates for the entropy (ϕ) are grouped in the first column, the coefficient of variation (C) in the second and the localisation measure (L) for $s = 0.50, 0.80$ are shown in the third and forth column of each table. We show estimates for hidden, displayed as well as total posted liquidity. Table 1.2 reports estimation of the average *price-dispersion* of liquidity, while table 1.3 reports estimates for the *time-dispersion* of liquidity.

The results of table 1.2 and 1.3 can be broadly summarised in three points. First, the estimates confirm the intuition that hidden liquidity is concentrated around few price quotes and few points in time. For instance, according to the L^{book} measure in table 1.2, on average 26% of price quotes already contain more than 80% of the hidden volume. The same degree of displayed liquidity is distributed across 80% of the price quotes on average.

Second, the difference in the degree of dispersion between hidden and displayed liquidity seems is larger in the time domain than in the price domain.

Third, we observe that hidden liquidity varies significantly more than displayed liquidity. Our unconditional coefficient of variation reports 2.72 for hidden and 0.73 for displayed liquidity in the price domain and 2.88 and 1.23 in the time domain.

Table 1.2: Estimates of localisation of hidden, displayed and total depth in the order book. Estimates are shown for four measures, Φ^{book}, C^{book}, $L^{book}_{0.50}$ and $L^{book}_{0.80}$

Liquidity	Φ^{book}			C^{book}			$L^{book}_{0.50}$			$L^{book}_{0.80}$		
Quintiles	*hidden*	*displayed*	*total*	*hidden*	*displayed*	*total*	*hidden*	*displayed*	*total*	*hidden*	*displayed*	*total*
q_1 (least)	0.88	0.97	0.97	2.45	0.82	0.92	0.09	0.24	0.24	0.30	0.54	0.54
q_2	0.87	0.97	0.97	2.61	0.77	0.87	0.09	0.26	0.25	0.30	0.55	0.56
q_3	0.86	0.97	0.97	2.69	0.72	0.84	0.08	0.26	0.26	0.26	0.56	0.56
q_4	0.84	0.98	0.97	3.04	0.69	0.88	0.06	0.28	0.26	0.23	0.57	0.57
q_5 (most)	0.85	0.98	0.98	2.81	0.66	0.78	0.06	0.29	0.27	0.22	0.59	0.59
all	0.86	0.97	0.97	2.72	0.73	0.86	0.08	0.27	0.26	0.26	0.56	0.56

Table 1.3: Estimates of localisation of hidden, displayed and total depth in time. Estimates are shown for four measures, ϕ^{time}, C^{time}, $L^{time}_{0.50}$ and $L^{time}_{0.80}$

Liquidity	Φ^{time}			C^{time}			$L^{time}_{0.50}$			$L^{time}_{0.80}$		
Quintiles	*hidden*	*displayed*	*total*	*hidden*	*displayed*	*total*	*hidden*	*displayed*	*total*	*hidden*	*displayed*	*total*
q_1 (least)	0.20	0.62	0.58	3.11	1.46	1.87	0.10	0.20	0.15	0.13	0.37	0.29
q_2	0.21	0.66	0.64	2.87	1.35	1.62	0.10	0.21	0.17	0.14	0.4	0.34
q_3	0.20	0.72	0.69	2.81	1.22	1.41	0.11	0.23	0.20	0.14	0.44	0.39
q_4	0.17	0.75	0.73	2.82	1.15	1.31	0.10	0.24	0.21	0.14	0.47	0.42
q_5 (most)	0.16	0.82	0.78	2.80	0.95	1.10	0.11	0.28	0.25	0.14	0.53	0.49
all	0.19	0.71	0.68	2.88	1.23	1.46	0.10	0.23	0.20	0.14	0.44	0.38

1.4.2 Impact of Hidden Order Submissions

In the preceding section, we have reported strong empirical evidence that hidden orders are submitted in large portions and by single investors. Naturally, the question arises what drives these hidden liquidity-spikes and whether these large chunks of hidden liquidity carry valuable information with respect to future returns. Are they related to informed trading? What motivates traders to issue large, hidden short-lived orders? And how does hidden liquidity affect the market overall? To address these issues, in this section we employ the event-study-framework as proposed in Campbell et al. (1997) and recently applied in a study on *quantifying market reactions to real-time news sentiment announcements*, see Gross-Klussmann and Hautsch (2010).

For this purpose, we first define events of significant imbalance-skew, i.e. *liquidity shocks*. We distinguish between displayed and hidden *order imbalances*. Therefore, let t_i denote the times at which we record the market and denote D_i^{bid}, H_i^{bid} and T_i^{bid} cumulated displayed, hidden and total depth at time t_i on the bid side of the order book and D_i^{ask}, H_i^{ask} and T_i^{ask}, respectively, for the ask side. Note that $D_i^{bid} + H_i^{bid} = T_i^{bid}$ holds. The respective displayed, hidden and total order imbalances are given as follows:

$$I_i^D = D_i^{bid} - D_i^{ask} \qquad I_i^H = H_i^{bid} - H_i^{ask} \qquad I_i^T = T_i^{bid} - T_i^{ask}. \tag{1.4.8}$$

Let us fix an imbalance threshold, say I. We call a time-point t_j an *event* of large hidden (displayed) excess imbalance, whenever the corresponding imbalance exceeds the critical value I, i.e. $I_i^H > I$ ($I_i^D > I$).

As we will have to consider cross-sectional comparison between different stocks, we normalise the choice of I and make it independent of the specific stock at hand. Therefore, given each stock's total imbalance I_i^T, we consider its p-quantile-function $F_T^{-1}(p)$.[9][10] Then, we can define a *normalised* threshold, i.e.

$$I_p := F_T^{-1}(p). \tag{1.4.9}$$

To calculate the event impact, we fix an imbalance quantile threshold p. Let $X = (X_{t_1}, X_{t_2}, ..., X_{t_m})$ denote some market quantity. Then, for some fixed time interval δ the impact of a liquidity shock on hidden and displayed depth can be quantified as follows

$$\overline{X}_{\delta,p}^D = E[X_{t_k+\delta} | I_{t_k}^D \geq I_p] \qquad \overline{X}_{\delta,p}^H = E[X_{t_k+\delta} | I_{t_k}^H \geq I_p] \quad . \tag{1.4.10}$$

$\delta > 0$ refers to the ex-post and $\delta < 0$ to the ex-ante impact. In the usual fashion, we conduct cross-sectional aggregation by weighted averages, where the weights equal the inverse of the respective sample variances. In other words, for stock k denote $\overline{X}_{\delta,p}^{D,k}$ and $\overline{X}_{\delta,p}^{H,k}$ its impact estimates according to (1.4.10). Denote $\sigma^2(\overline{X}_{\delta,p}^{D,k})$ and $\sigma^2(\overline{X}_{\delta,p}^{H,k})$ its respective variances. Then cross-sectional means are obtained as follows:

$$\overline{X}_{\delta,p}^D = \frac{\sum_{k=1}^N \sigma^{-2}(\overline{X}_{\delta,p}^{D,k}) \overline{X}_{\delta,p}^{D,k}}{\sum_{k=1}^N \sigma^{-2}(\overline{X}_{\delta,p}^{D,k})} \qquad \overline{X}_{\delta,p}^H = \frac{\sum_{k=1}^N \sigma^{-2}(\overline{X}_{\delta,p}^{H,k}) \overline{X}_{\delta,p}^{H,k}}{\sum_{k=1}^N \sigma^{-2}(\overline{X}_{\delta,p}^{H,k})}. \tag{1.4.11}$$

In the sequel, we will analyse the impact on realised volatility, spread, depth and *cumulative abnormal return* CAR.

[9] The quantile function is defined as $F^{-1}(p) := \inf\{x \in \mathbb{R}_+ : F(x) \geq p\}$, where F denotes the cumulative distribution of x.

[10] By considering the total instead of the displayed or hidden imbalances for identifying the threshold imbalances, we reduce the hidden/displayed selection bias.

Cumulative Abnormal Returns

In defining abnormal returns, we follow closely Campbell et al. (1997). As a model for stock $k's$ "normal" returns we assume the following *market model*:

$$R^k_{t_i} = \alpha_k + \beta_k R^{market}_{t_i} + \epsilon^k_{t_i}, \tag{1.4.12}$$

with $\epsilon^k_{t_i}$ normal and iid random variables. R^{market}_t is the so called *market return*, and stock i's actual return R^i_t.[11] Our choice for the market model will be the S&P 500 index return. In order to derive CAR, first we estimate (1.4.12), based on the 1-minute-snapshot NASDAQ ModelView dataset. Estimation is done without including the event windows. Provided with the parameter estimates α_k and β_k, we may again use (1.4.12) to compute the single abnormal returns according to $\widehat{R}^k_{t_i} = R^k_{t_i} - \widehat{\alpha}_k - \widehat{\beta}_k R^{market}_{t_i}$. Now, starting at some time t_i, the k^{th} stock *cumulative abnormal return* up to some later time $t_i + \delta$ reads

$$\widehat{CAR}^k_{i\delta} = \prod_{t_i \le t_l \le t_i + \delta} (\widehat{R}^k_{t_l} + 1) - 1. \tag{1.4.13}$$

Impact Estimates

Results are shown in the figures 1.2-1.5 for cumulative excess returns, volatility, spread and depth. To test for generic effects, we report estimates for varying degrees of imbalance I_p, i.e. $p = 60\%, 70\%, 80\%, 90\%$. We henceforth refer to the event when order imbalance exceeds the critical imbalance threshold I_p a *large order imbalance shock* (LOS). Impact estimates have been conducted for $\delta = -60\ min$ (one hour pre-LOS) to $\delta = 120\ min$ (two hours post-LOS). The spread, volatility and depths are normalized by their unconditional means.

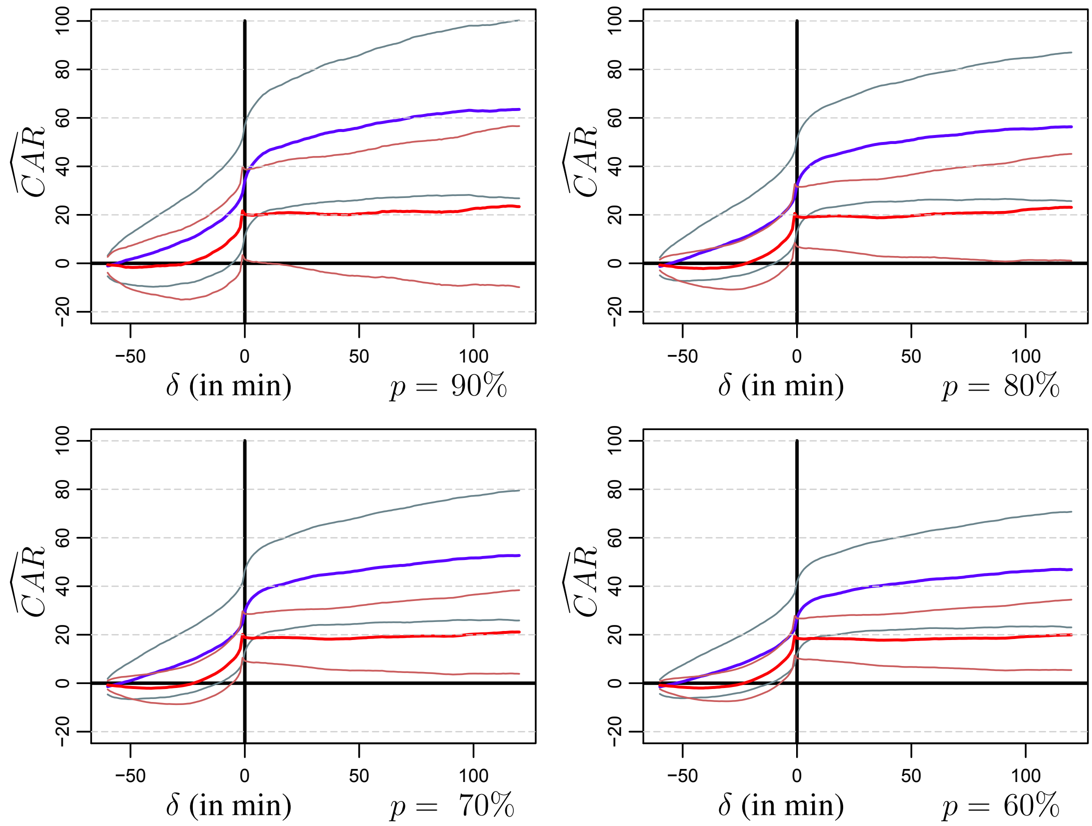
The result of hidden (displayed) liquidity shocks is given in blue-coloured (red-coloured) solid lines. Light-coloured lines correspond to the upper and lower $95\%-$ confidence intervals based on normality assumption. To check for statistical significance, thick black lines provide the unconditional standard deviation of the spread itself.

The results can be summarised in several points. First, the ex-post return impact of large hidden orders is highly significant, whereas it is insignificant for displayed shocks. For instance, high hidden imbalances (i.e. $p = 90\%$) generate an ex-post return impact of approximately 35 basis points. In comparison, the same imbalance in displayed depth generates only minor 3.5 basis points. The return impact for hidden imbalances even exceeds by far the impact of relevant earnings announcements news (c.f. Gross-Klussmann and Hautsch (2010)).

Second, large hidden orders herd with trending markets, i.e. positive excess returns induce large buyers to submit large buy hidden orders. This effect is less significant for traders who use displayed orders. This might be due to momentum trading or due to liquidity trading. Large trades that get executed over an extended time period induce a serial correlation in returns and order flows. The fact that large hidden orders get submitted in markets that show a trend indicates that large investors use large hidden orders to minimise price impact. Two separate

[11] Typically R^{market} is chosen to be some predictor for the stock's actual return. For instance, in the most simple case, one may choose the stock's expected return.

Figure 1.2: Impact of hidden and displayed order imbalances on the cumulated abnormal return $\widehat{CAR}$. Results are reported for varying degrees of imbalances p.



observations reinforce the liquidity trader perspective. First, the ex-post return impact shows a concave square-root law performance. Price impact of (market) orders is known to exhibit this kind of pattern in time (c.f. Farmer and Lillo (2003)). Second, the fact that the impact grows with the size of the imbalance suggests that price shifts are due to price-size effects.

Third, large orders get submitted on one side of the market, when the spread is narrow. Cross-sectionally, the spread is more than 10% below its unconditional average before/at submission of large hidden or displayed orders. This is in line with the fact that narrow spreads make it more attractive for liquidity demanders to submit market orders. Liquidity suppliers anticipate this behaviour and provide more liquidity. The submission of hidden and displayed orders differ in their speed. While hidden orders get submitted almost instantaneously upon decline in spread, displayed orders get submitted approximately 10-15 after the spread declines.

Fourth, the presence of large hidden orders is associated with large total depth. This suggests that hidden orders are used to reduce the costs of liquidity competition when competition among liquidity suppliers is high.

Sixth, submission of large hidden or displayed orders is not associated with a significant change in conditional volatility.

1.4.3 Discussion: The Downside of Hidden Liquidity

Two observations afford a more detailed discussions. First, hidden orders are associated with significant ex-post returns and second, we find strong evidence that this is not due to informed trading; i.e. the results indicate that the price movements are in line with liquidity effects.

Hidden Liquidity Bypasses Liquidity Demanders

In line with Cebiroglu et al. (2012), our findings indicate that hidden orders induce price inefficiencies, i.e. price variations that are not due to changes in the fundamental value. In particular, hidden orders unwittingly bypass potentially large pools of *latent* liquidity by not properly exposing trade interest. As a consequence, the increased risk of counterparty mismatch does also materialise into higher cancellation rates. Since eventually, cancelled hidden orders need to be liquidated via *costly* market orders, hidden orders generate higher price pressures and price fluctuations.

Do Hidden Orders Increase Volatility?

The fact that hidden order submissions induce an ex-post increase in price changes also suggests that hidden order submissions increase market volatility. However, we find that when volatility is conditioned on order arrival, there is no increase in ex-post volatility as of figure 1.3. This indicates two things: 1) the information that hidden orders have arrived eliminates most of the random variation and 2) the ex-post price impact of hidden orders is mostly deterministic.

To see that the conditional variance is not affected by the submission of the order, while the unconditional variance is affected, consider an order of size N that arrives with probability p at time t. We assume a simple market impact model: the price impact is linear in the order size N, i.e. the return between t and $t+1$ reads

$$r_{t,t+1} = \begin{cases} \alpha N + \epsilon & \text{if} \\ \epsilon \end{cases} \tag{1.4.14}$$

with ϵ independent of the order arrival and normal with variance σ^2 and mean μ. According to the law of binomial random variables, the unconditional variance of the return reads

$$Var[r_{t,t+1}] = \sigma^2 + \alpha^2 N^2 p(1-p). \tag{1.4.15}$$

On the other hand, $Var_t[r_{t,t+1}]$ variance conditioned at time t - i.e. after arrival of the order - reads

$$Var_t[r_{t,t+1}] = \sigma^2 < Var[r_{t,t+1}] \tag{1.4.16}$$

Hence, if orders induce a deterministic drift then they also increase (unconditional) market volatility.

1.5 Conclusion

Since public exchanges have a crucial role in the price discovery process, the role of pre-trade transparency is highly relevant. However, although the proliferation of hidden liquidity has seen a dramatic increase over the recent years and has increasingly shifted to the centre of regulatory debates, the discussion about its benefits and costs is still far from being conclusive. Partly due to data limitation, only few studies have succeeded in examining the characteristics and implications of hidden liquidity on public exchanges. Most studies analyse hidden orders on a single order basis. Using the unique NASDAQ ModelView data, our study empirically examines the determining conditions under which markets supply more hidden liquidity for a large sample of 448 stocks from the S&P 500 index, traded during September 2008 to March 2009.

We analyse and identify the main determinants of posted and traded hidden liquidity for the S&P500. We find that observable stock characteristics well explain the cross-sectional presence of hidden liquidity. Using a model-selection approach, we identify the stock's average spread as the main determinant for the presence of hidden liquidity.

We find that these individual submissions carry significant impact on post submission returns. The price impact is persistent and can exceed the information content of news related to important earnings announcements. Our findings strongly support the *counterparty attraction* hypothesis as developed in Cebiroglu et al. (2012). Accordingly, price shifts are not due to information arrival but due to trading frictions and (il-) liquidity effects: since large hidden orders do not attract counter-parties and are more likely to get cancelled, investors need to trade larger portion of their shares via costly market orders. The increased portion in market order submission generates price pressures and causes price fluctuations. In other words, hidden orders can be the bearer of price inefficiencies and market frictions.

Our findings have important implications for exchange operators and policy makers alike. A good assessment on the implications of hidden order submissions and hidden liquidity is important to establish proper pre-trade transparency rules, ensure fair market competition and reduce price inefficiencies. Separately, as liquidity discovery is an integral part of an efficient trading process, investors, portfolio managers and trading desks are interested in exploring new undetected pools of liquidity. In that respect, our analysis provides evidence on how to locate presence of hidden liquidity based on the knowledge of observable and readily available stock characteristics.

Appendix 1.A Descriptive Statistics

Table 1.4: Crosscorrelation estimates among observable stock characteristics. We report cross-correlations among average daily traded volume (ADV), the High-Low variation ($HiLo$), the spread ($Spread$), price ($Price$) and the top-of-book depth (Top).

	ADV	$Time$	$HiLo$	$TrSize$	$Spread$	$Price$	Top
ADV	$*$	-0.570	0.248	0.584	-0.382	-0.213	0.529
$Time$		$*$	-0.267	-0.298	0.529	0.176	-0.218
$HiLo$			$*$	0.463	-0.146	-0.403	0.287
$TrSize$				$*$	-0.351	-0.490	0.941
$Spread$					$*$	0.698	-0.285
$Price$						$*$	-0.420
Top							$*$

Table 1.5: Cross-sectional sample averages for observable stock characteristics and hidden liquidity. We report averages for the average daily traded volume (ADV), the inter-arrival time of trades $time$, the trade size $size$, the spread $spread$, the mid-point price $price$ and the visible depth at the top (first level) of the book (D^{top}) and the total *posted* hidden volume H and ratio with respect to total depth D, as well as the total *traded* hidden volume H_{traded} and and its ratio with respect to total traded volume (ADV). Cross-sectional averages are grouped according to their liquidity quintiles based on ADV. On a daily basis, $HiLo$ is computed as the daily high-low difference in proportion to the prevailing daily mid-point price.

Liquidity Quantile	Observable Stock Properties							Hidden Liquidity posted		Hidden Liquidity traded	
	ADV ($10^6 sh.$)	$time$ ($sec.$)	$HiLo$ ($ratio$)	$size$ ($sh.$)	$spread$ ($ticks$)	$price$ (\$)	D^{top} ($sh.$)	H ($sh.$)	H ($ratio$)	H_{traded} ($10^6 sh.$)	H_{traded} ($ratio$)
q_1 (least)	1.39	2.65	0.07	147	4.91	36.46	308	656	0.19	0.37	0.26
q_2	2.72	1.38	0.08	158	3.39	32.84	576	1318	0.20	0.57	0.20
q_3	4.23	0.94	0.09	165	2.40	27.41	800	1671	0.17	0.69	0.15
q_4	7.13	0.61	0.10	178	1.87	24.59	1278	2292	0.16	0.83	0.11
q_5 (most)	16.98	0.35	0.11	219	1.38	23.32	3490	6202	0.13	1.10	0.07
all	6.57	1.19	0.09	174	2.79	28.91	1305	2440	0.17	0.71	0.16

Table 1.6: Sample statistics on hidden liquidity sorted by liquidity quintiles. The table reports the cross-sectional mean, maximum (Max), minimum (Min) and standard deviation (SD) for the posted hidden volume without the spread, i.e. "deep" in the book (H_B), in the spread (H_S), the total posted hidden liquidity (H), the total posted displayed liquidity (D) and the total posted liquidity ($Tot := D+H$).

Liquidity Quantile	Mean (in 1000 sh.)					Median (in 1000 sh.)					Max (in 1000 sh.)					Min (in 1000 sh.)					SD (in 1000 sh.)				
	H_B	H_S	H	D	Tot	H_B	H_S	H	D	Tot	H_B	H_S	H	D	Tot	H_B	H_S	H	D	Tot	H_B	H_S	H	D	Tot
q_1 (least)	0.78	0.11	0.89	4.2	5.1	0.20	0.01	0.28	3.7	4.3	39	15	41	29	53	0	0	0	0.01	0.02	2.14	0.49	2.25	2.65	3.80
q_2	1.24	0.12	1.36	6.5	7.8	0.45	0.01	0.52	5.9	6.9	60	25	63	43	79	0	0	0	0.03	0.04	3.2	0.68	3.4	3.7	5.3
q_3	1.80	0.11	1.90	12	14	0.59	0.01	0.65	11	12	83	27	85	73	120	0	0	0	0.09	0.12	4.8	0.67	4.9	6.4	8.8
q_4	2.29	0.12	2.41	14	17	0.55	0.01	0.62	14	15	143	33	145	84	173	0	0	0	0.04	0.07	7	0.82	7.1	7.7	11
q_5 (most)	5.80	0.10	5.80	45	51	1.53	0.00	1.61	43	47	229	50	231	273	367	0	0	0	0.33	0.50	15	1.03	15	24	30
all	2.37	0.11	2.48	16	19	0.66	0.01	0.74	15	17	111	30	113	100	159	0	0	0	0.10	0.15	6.3	0.74	6.5	8.8	12

Appendix 1.B Hidden Liquidity Determinants

Table 1.7: Parameter estimates of the cross-sectional regression models for traded and posted hidden liquidity according to the models (1.3.1) to (1.3.8). T-statistics are reported in round brackets. We report *adjusted* r^2, f-statistic and respective gains in percentages with respect to the standard (i.e. non-transformed) models (1.3.1) to (1.3.4).

Variable	Posted Hidden Liquidity				Traded Hidden Liquidity			
	Volume		Ratio		Volume		Ratio	
	standard (1.3.1)	*transformed* (1.3.5)	*standard* (1.3.2)	*transformed* (1.3.6)	*standard* (1.3.3)	*transformed* (1.3.7)	*standard* (1.3.4)	*transformed* (1.3.8)
$Intercept$	$-2.51e+03$	$6.06e+00$	$-7.37e-01$	$1.62e+00$	$-2.89e+06$	$1.63e+01$	$2.23e-02$	$6.22e-01$
$(t-statistic)$	(-1.10)	(10.70)	(-3.17)	(3.2)	(-1.56)	(48.37)	(1.06)	(24.6)
ADV	$5.95e-05$	$1.92e-08$	$-1.18e-08$	$-4.43e-09$	$3.59e-01$	$4.02e-08$	$-2.65e-09$	$-9.86e-10$
$(t-statistic)$	(1.41)	(3.34)	(-2.75)	(-0.87)	(10.49)	(11.77)	(-6.82)	(-3.84)
$Time$	$-3.78e+02$	$-2.38e-01$	$-1.98e-01$	$-2.68e-01$	$-7.08e+05$	$-4.18e-01$	$2.42e-02$	$4.79e-03$
$(t-statistic)$	(-1.19)	(-5.47)	(-6.09)	(-6.93)	(-2.75)	(-16.17)	(8.23)	(2.47)
$HiLo$	$-1.92e+04$	$-2.00e+00$	$1.06e+00$	$1.53e+00$	$5.60e+07$	$2.95e+00$	$2.91e-01$	$-5.41e-02$
$(t-statistic)$	(-2.59)	(-2.00)	(1.41)	(1.72)	(9.33)	(4.95)	(4.26)	(-1.21)
$TrSize$	$3.33e+01$	$7.68e-03$	$5.04e-03$	$1.02e-02$	$-8.57e+03$	$2.62e-03$	$-5.56e-05$	$1.59e-04$
$(t-statistic)$	(2.12)	(3.7)	(3.15)	(5.52)	(-0.67)	(2.12)	(-0.38)	(1.72)
$Spread$	$2.66e+04$	$-9.88e-02$	$1.65e+01$	$1.13e+00$	$-7.96e+07$	$3.97e-01$	$1.31e+00$	$1.29e-01$
$(t-statistic)$	(1.58)	(-0.98)	(9.62)	(12.54)	(-5.84)	(6.62)	(8.43)	(28.62)
$Price$	$-2.79e+01$	$-9.38e-03$	$9.39e-03$	$1.11e-02$	$2.59e+05$	$1.39e-02$	$2.61e-03$	$4.06e-04$
$(t-statistic)$	(-1.36)	(-2.93)	(4.48)	(3.89)	(15.57)	(7.31)	(13.79)	(2.84)
Top	$7.85e+01$	$2.52e-03$	$-7.14e-03$	$-1.53e-02$	$-5.84e+04$	$-1.72e-02$	$-2.72e-04$	$-5.01e-04$
$(t-statistic)$	(3.07)	(0.75)	(-2.73)	(-5.13)	(-2.81)	(-8.63)	(-1.15)	(-3.34)
$f-statistic$	55	124	69	147	124	262	360	978
$(Gain)$	$(*)$	(+126%)	$(*)$	(+113%)	$(*)$	(+112%)	$(*)$	(+172%)
r^2	0.457	0.657	0.515	0.695	0.657	0.803	0.849	0.939
$(Gain)$	$(*)$	(+44%)	$(*)$	(+35%)	$(*)$	(+22%)	$(*)$	(+11%)

Table 1.8: Estimates of the LARS procedure for posted hidden liquidity. For each LARS-step we report the choice of the selected variable ("'action"'), the current goodness-of-fit in terms of pure r^2_{lars} and relative to the full model r^2_0 as reported in table 1.7 and finally the Akaike information Criteria C_p.

	Posted Hidden Volume								Posted Hidden Ratio							
Step	Volume				Volume (Transformed)				Ratio				Ratio (Transformed)			
	$action$	r^2_{lars}	$\frac{r^2_{lars}}{r^2_{ols}}$	C_p	$action$	r^2_{lars}	$\frac{r^2_{lars}}{r^2_{ols}}$	C_p	$action$	r^2_{lars}	$\frac{r^2_{lars}}{r^2_{ols}}$	C_p	$action$	r^2_{lars}	$\frac{r^2_{lars}}{r^2_{ols}}$	C_p
1	Top	0.34	0.73	99	$TrSize$	0.30	0.46	464	$Spread$	0.18	0.34	316	$Spread$	0.29	0.41	601
2	$TrSize$	0.44	0.94	22	ADV	0.37	0.56	376	$Price$	0.40	0.77	108	$Price$	0.55	0.78	219
3	ADV	0.45	0.98	9	$Time$	0.47	0.71	249	$Time$	0.41	0.78	105	$HiLo$	0.59	0.85	159
4	$HiLo$	0.46	0.99	6	$Price$	0.60	0.90	84	$HiLo$	0.50	0.96	23	$Time$	0.67	0.96	42
5	$Time$	0.46	0.99	7	Top	0.65	0.97	27	ADV	0.51	0.97	18	$TrSize$	0.68	0.97	35
6	$Spread$	0.46	0.99	9	$HiLo$	0.65	0.98	20	$TrSize$	0.51	0.98	16	Top	0.70	0.99	12
7	$Price$	0.47	1.00	8	$Spread$	0.66	1.00	8	Top	0.52	1.00	8	ADV	0.70	1.00	8

Table 1.9: Estimates of the LARS procedure for traded hidden liquidity. For each LARS-step we report the choice of the selected variable ("‘action"’), the current goodness-of-fit in terms of pure r^2_{lars} and relative to the full model r^2_0 as reported in table 1.7 and finally the Akaike information Criteria C_p.

	Executed Hidden Volume								Executed Hidden Ratio							
Step	Volume				Volume (Transformed)				Ratio				Ratio (Transformed)			
	$action$	r^2_{lars}	$\frac{r^2_{lars}}{r^2_{ols}}$	C_p	$action$	r^2_{lars}	$\frac{r^2_{lars}}{r^2_{ols}}$	C_p	$action$	r^2_{lars}	$\frac{r^2_{lars}}{r^2_{ols}}$	C_p	$action$	r^2_{lars}	$\frac{r^2_{lars}}{r^2_{ols}}$	C_p
1	$Price$	0.05	0.08	796	$Price$	0.08	0.09	1657	$Spread$	0.35	0.41	1476	$Spread$	0.92	0.98	176
2	$Time$	0.21	0.32	585	$Time$	0.33	0.41	1072	$Price$	0.56	0.66	849	$TrSize$	0.92	0.98	167
3	ADV	0.40	0.60	343	Top	0.57	0.7	544	ADV	0.69	0.81	468	ADV	0.93	0.99	78
4	Top	0.42	0.63	322	ADV	0.59	0.73	503	$Time$	0.72	0.84	393	Top	0.94	0.99	74
5	$HiLo$	0.61	0.92	70	$HiLo$	0.73	0.9	189	$TrSize$	0.79	0.92	199	$Price$	0.94	0.99	70
6	$Spread$	0.66	1.00	8	$Spread$	0.80	1.00	11	Top	0.84	0.99	32	$Time$	0.94	1.00	23
7	$TrSize$	0.66	1.00	8	$TrSize$	0.81	1.00	8	$HiLo$	0.85	1.00	8	$HiLo$	0.94	1.00	8

Appendix 1.C Hidden Liquidity Impact

Figure 1.3: Impact of Hidden Order Imbalances on the realized 10-minute volatility. Results are reported for varying degrees of hidden (displayed) order imbalances p. Volatility has been normalized by its unconditional, historical mean.

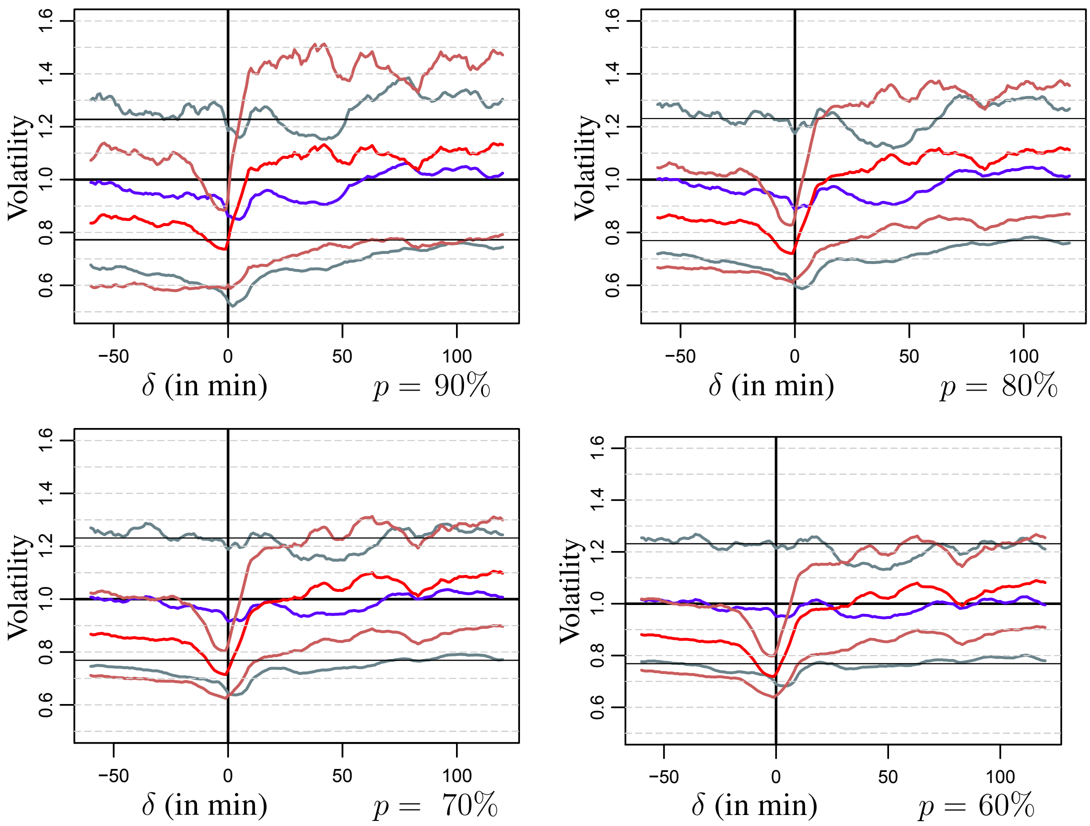


Figure 1.4: Impact of hidden order imbalances on the spread. Results are reported for varying degrees of hidden (displayed) order imbalances p. Spread has been normalized by its unconditional, historical mean.

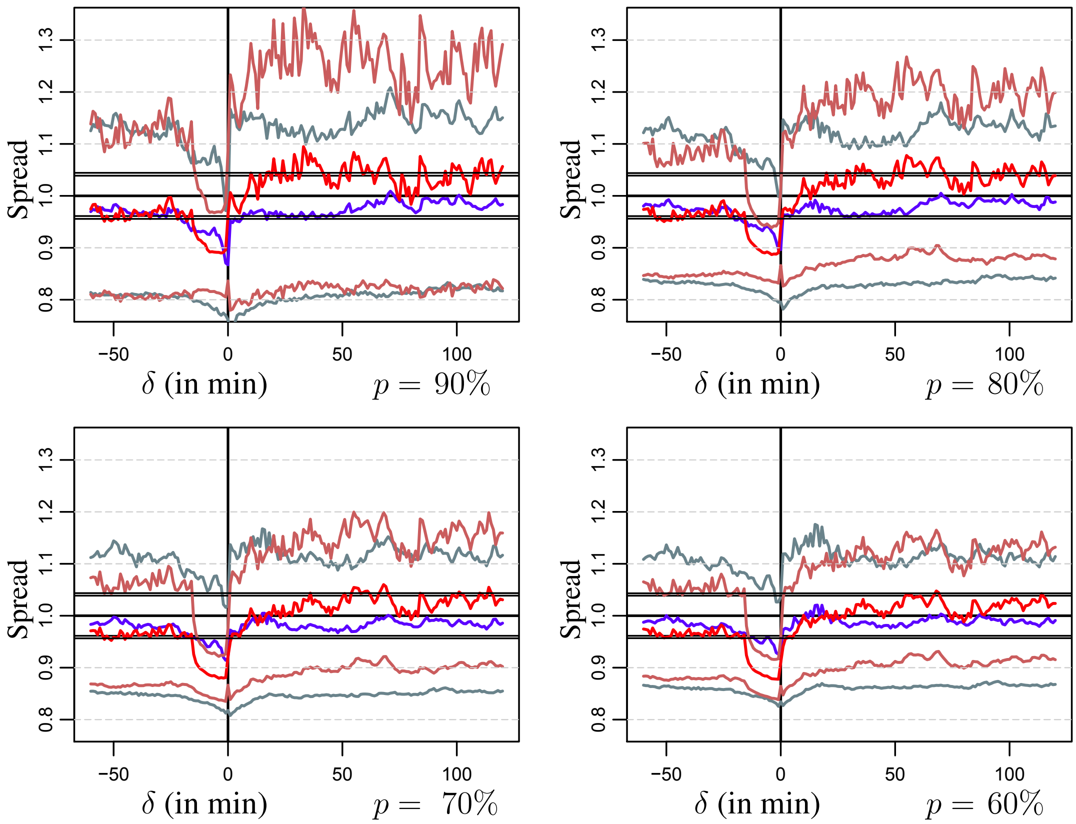
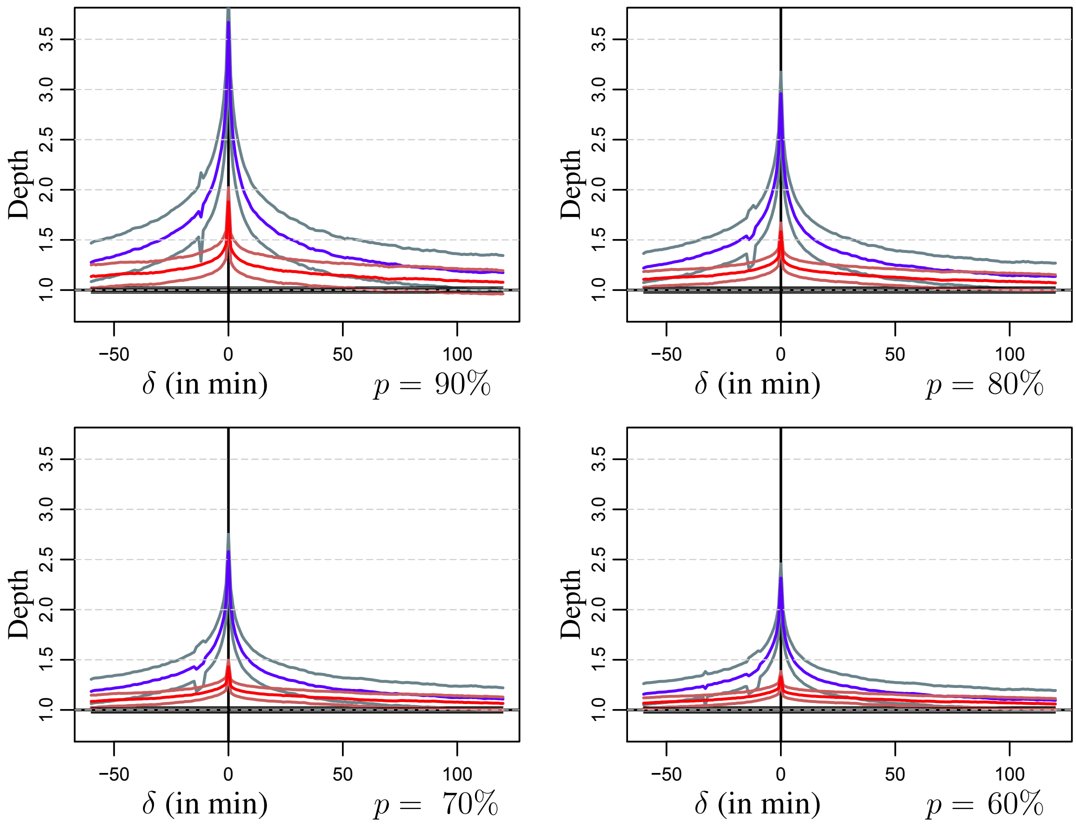


Figure 1.5: Impact of hidden and displayed order imbalances on total order book depth D. Results are reported for varying degrees of hidden (displayed) order imbalances p. Total depth is normalized by its unconditional mean.



Chapter 2

Optimal Order Exposure and the Market Impact of Limit Orders: A Structural Model

This chapter is based on Cebiroglu and Horst (2013).

2.1 Introduction

The use and proliferation of *hidden liquidity* among the major stock exchanges has considerably increased over the recent years. Nowadays, hidden orders, *Iceberg orders* or so called *reserve orders* have become prevalent features of modern electronic markets.[2] Yet, exchanges still require openly displayed quotes to effectively organize trade. In particular, by giving displayed orders higher *execution-priority* than hidden orders, typical *order-driven* or *limit-order markets* actively encourage their market participants to openly display their quotes. Thus, exposing trade demand lies at the basis of markets itself. On the other hand however, as *order exposure* is inherently associated with *information-leakage*, there is a need to control exposure. Indeed, literature suggests that disclosure of trade interest can be risky; *adverse selection*, the risk of getting *quote-matched* or *front-run*, they all can increase transaction costs. Thus, although exposure can lead to higher execution-priority, it comes at the cost of increased exposure-risk. Traders need to balance these two antagonistic sources of risk and the optimal exposure ultimately involves a trade-off.

[2]A growing body of empirical studies indicate the wide-spread use of *hidden* orders. For instance, Pascual Gasco and Veredas (2008) report that 26% of all trades on the Spanish Stock Exchanges involve hidden volume. Frey and Sandas (2009) report that 9.3% of *submitted* and 15.9% of *executed* shares contain Iceberg orders on the German Xetra Stock Exchange.[3] Further studies confirm that hidden liquidity is particularly prevalent among large investors: D'Hondt et al. (2004) report that 81% of orders with total sizes in the largest quartile are Icebergs or (partly) hidden orders. Supplementing this findings, Frey and Sandas (2009) find that Iceberg orders are on average 12-20 times larger than limit orders.

In this paper, we propose a structural model that captures this trade-off and derive implications for exposure strategies. In particular, we assess how *market impact of limit orders* materialises. To our best knowledge, we provide the first estimates of optimal display sizes using high-frequency order-message data.

We consider a single risk-neutral trader who liquidates a large portfolio over a fixed trading horizon. The trader has a pre-specified *reference* or *benchmark price* and the additional freedom to hide a portion of the order. Order execution is governed by order arrivals and cancellations. In order to keep the analysis tractable and transparent, we do not model individual order arrivals but rather introduce *aggregate* limit and market order flow volumes and cancellation ratios that add and retract liquidity on the trader's side of the market. The random flows determine the execution volume at terminal time; assuming a liquidation constraint at the end of the trading period, unexecuted orders are executed against the best prevailing opposite price. The price process is modelled as a separate (independent) stochastic process. The trader's goal is to determine the optimal exposure (display size) so as to minimise his transaction costs.

The trader is faced with liquidity competition and liquidity demand that may be affected by its choice of display size, i.e. *exposure* or *market impact*. The assumption that the trader's exposure decision impacts the market dynamics, especially incoming order flows, is central to our model. It is mainly build on the empirical observation that the state of the order book has informational value with respect to its future state.[4]

Our framework is amenable to mathematical analysis. We find an explicit representation of the trade-off between exposure costs and benefits and derive optimal exposure strategies under different model specifications. This allows to link exposure decisions to the state of the market and provide a microstructural rationale for the presence of hidden liquidity in limit order markets.

The main results can be summarised as follows. First, when the investor is small (or the market is liquid), adverse effects that arise from exposure become negligible. This follows from the fact that small investors do not have the size to significantly alter the shape of the book. Hence, in line with empirical findings (c.f. Bessembinder et al. (2009)), we predict that hidden liquidity is used less in liquid markets. However, when investors are not small, adverse exposure affects can be substantial and investors have to hide an increasing portion of their order.

Second, when the investor is very large, we find that the decision to hide or display does only depend on price impact, but not on order flow impact. This follows, since large orders have a lower chance of execution, the transaction costs are mostly driven by *opportunity costs*, i.e. by the terminal market price. In particular, in markets where prices drift away from (move towards) the excess side of order imbalance, large investors hide (display) their full trading intentions.

Third, the initial state of the order book plays a critical role in the decision to hide or expose trading interests. When order imbalances are extremely skewed to one side of the market, traders are better off displaying their intentions. The explanation is clear. Adding orders to an already highly skewed market is not going to significantly alter the state of the book and market dynamics

[4]For (empirical) evidence, see Biais et al. (1995), Ranaldo (2004), Griffiths et al. (2000).

Fourth, we identify market specifications under which the execution performance does not depend on the order size at all. This holds when liquidity competition at the submission price level is low and exposure impact on the supply and demand side of liquidity fulfil a balance condition. Hence, an increase (decrease) in liquidity competition is offset by a proper increase (decrease) in liquidity demand and vice versa.

Fifth, liquidity competition reacts stronger than liquidity demand and increases with exposure size.

We estimate model and market impact parameters using high-frequency order message data as provided by the INET exchange. Parameters are estimated for a range of stocks from the S&P 500 for different high-frequency periods (i.e. 3sec., 10sec. and 30sec.) and different order book states. These estimates allow to recover optimal display sizes under different market specifications. Our analysis shows that on high-frequency time scales, hidden orders can significantly increase trading performance, particularly when the order book imbalance is already skewed towards the same-side of the market.

Our structural approach is similar to Hollifield et al. (2006). However, their focus is the trade-off between price and execution risk. As we focus on the exposure effect, we abstract from this mechanism. In particular, price is not a choice variable in our model. There are also several distinguishing marks with respect to the existing theoretical literature on hidden orders, especially the equilibrium models of Buti and Rindi (2013) and Moinas (2010) and the optimal liquidation framework in Esser and Mönch (2007). First, we do not assume asymmetric information as Buti and Rindi and Moinas do. Besides the fact that this feature is difficult to reconcile with high-frequency data, there are reasons to believe that trading frictions do not derive from informational asymmetry alone.[5] Second, Buti and Rindi (2013) and Moinas (2010) enforce strong restrictions on the order size, i.e. transaction sizes take on a finite set of values. In fact, order flow volume distributions on high-frequency time scales are generically governed by zero-augmented point-mass mixture distributions (c.f. Hautsch et al. (2013)). By incorporating this structural element, our model provides a more realistic assessment of incoming order flow. Third, our concept of exposure impact is general and flexible enough to account for a wide range of exposure-impact scenarios. For instance, Buti and Rindi (2013) do not account for exposure effects on the demand side of liquidity (i.e. scaring away) and Moinas (2010) does not capture effects on the supply side of liquidity (i.e. front-running). Esser and Mönch (2007) assumes exposure effects only on prices, not on order flows.[6]

Our work also complements the study of *the market impact of limit orders*. For instance, Hautsch and Huang (2012) have shown that exposed limit orders affect future prices. We extend this line of research to the order flow dimension: we assess how order exposure affects both the supply and the demand side of liquidity. In particular, we find that incoming limit order flow reacts more strongly to order exposure at the best levels than market order flow.

[5]For empirical evidence, see Madhavan et al. (1997) and Huang and Stoll (1997).

[6]This model extends the model developed in Cebiroglu (2009) in two ways. To account for more realistic features of the high-frequency order book dynamics, this model assumes an additional positive mass at zero for the order flow distribution. Secondly, the hidden depth at the submission price level is assumed to be random.

The remainder of this paper is structured as follows. In Section 2.2, we introduce the model, including the order flow and price dynamics and calculate the respective objective function. We derive various theoretical results with respect to optimal expsure under different market specifications. In Section 2.3, we estimate the model (market impact) parameters and provide estimates for the optimal exposure size for various stocks and market settings. We benchmark the Iceberg performance against the fully-displayed limit order. Section 2.4 concludes.

2.2 The Model

We consider an Iceberg order trader ("she"), who trades for *liquidity* reasons. Specifically, the trader aims to buy a fixed (large) position of N shares over a (short) trading period $[0, T]$. Her *reference price* is the prevailing best bid price (B_0) at which she submits her order. The trader can choose to openly display any number $\Delta \in [0, N]$ of shares in the order book. The remaining $N - \Delta$ shares are shielded from public view and remain hidden until execution or cancellation. A random number Z^Δ of shares is executed before the end of the trading period. In order to enforce full liquidation at the end of the trading period, the unexecuted part $N - Z^\Delta$ of the order is cancelled at the terminal time T and executed against standing sell limit orders at the then prevailing best ask price A_T^Δ. For simplicity, we assume that market orders incur no transaction costs. The impact of market orders has already been extensively studied in the recent literature (c.f. Almgren and Chriss (1999), Alfonsi et al. (2010)). The dependence of the execution volume Z^Δ and best ask price A_T^Δ on the display size Δ accounts for the possible *impact* visible orders have on the dynamics of the order book. The *absolute* transaction costs are given by

$$\widetilde{P}^\Delta := Z^\Delta B_0 + \left(N - Z^\Delta\right) A_T^\Delta.$$

To facilitate performance comparisons across assets, we consider *relative* transaction costs (i.e. implementation shortfall) and chose the *relative execution price* as our performance measure. We define the relative execution price P^Δ as the difference between the average trade price per share and the time-of-trade quotation normalised by the submission price B_0; for $S_T^\Delta = (\widetilde{S}_T^\Delta\ )/B_0 := (A_T^\Delta - B_0)/B_0$

$$P^\Delta := \frac{\widetilde{P}^\Delta - NB_0}{NB_0} = \frac{\left(N - Z^\Delta\right)\left(A_T^\Delta - B_0\right)}{NB_0} = \frac{\left(N - Z^\Delta\right)\widetilde{S}_T^\Delta}{NB_0} = \left(1 - \frac{Z^\Delta}{N}\right) S_T^\Delta, \tag{2.2.1}$$

where the term $(1 - \frac{Z^\Delta}{N})$ represents the unexecuted proportion of the Iceberg order, and S_T^Δ represents the relative difference between the benchmark and submission price B_0 and the realised price A_T^Δ of the unexecuted part (i.e. *effective spread*).

On short time horizons, the bid and ask side of the market are not tightly following each other as evidenced by the well known *widening* and *narrowing* of the spread. For simplicity, we assume that both sides are independent, i.e. the execution volume in the buy-side of the market

Z^{Δ} and the best ask price A_T^{Δ} are conditionally independent with respect to Δ. For fixed Δ, the *expected relative execution price* reads

$$\begin{aligned}
W(\Delta) &:= E\left[P^{\Delta}\right] \\
&\overset{(2.2.1)}{=} E\left[\left(1-\frac{Z^{\Delta}}{N}\right)\cdot S_T^{\Delta}\right] \\
&\overset{(*)}{=} E\left[\left(1-\frac{Z^{\Delta}}{N}\right)\right]\cdot E[S_T^{\Delta}] \\
&= \left(1-\frac{E[Z^{\Delta}]}{N}\right)\cdot \mu(\Delta),
\end{aligned} \tag{2.2.2}$$

where $\mu(\Delta) := E[S_T^{\Delta}]$ denotes the expected effective spread at terminal time and we may denote the expected execution volume henceforth by $V := E[Z^{\Delta}]$. Conditional independence is used in $^{(*)}$. Note that for fixed Δ, conditional and unconditional expectations are the same. The trader's problem is to find the display size Δ^* that minimises the expected relative execution costs, i.e. the *implementation shortfall* (c.f. Perold (1988)).

Definition 1. *The optimal display size Δ^* is defined as*

$$\Delta^* = \underset{0\le\Delta\le N}{\arg\min}\left\{W(\Delta)\right\}. \tag{2.2.3}$$

In order to guarantee that an optimal display size exists it is sufficient to assume some form of continuity of the dependence of the distribution of the total execution volume and ask price on the displayed part of the Iceberg order.

2.2.1 Order Arrival Dynamics and Execution Priority

The executed iceberg order volume is determined by the incoming order flow. Sell market orders execute against standing buy limit orders and improve the chance of execution while incoming buy limit orders add liquidity to the same side of the book and hence impede the chance of execution. Modelling the full dynamics of *individual* order arrivals and cancellations would render the analysis of our model too complex. To enhance tractability, we use a *reduced-form model* of *aggregate* order flow. Specifically, order flows are aggregated into single submissions, effectively reducing our model to a 2-stage model: first (*aggregate*) limit orders arrive (or cancel); subsequently (*aggregate*) market orders arrive.

Orders arrive according to a probabilistic dynamic that is independent of the stock price process; the aggregate market order volume arriving during the period $[0, T]$ is denoted $x \geq 0$ while the aggregate limit order volume at the submission and more competitive price levels is denoted $y \geq 0$ and $\hat{y} \geq 0$, respectively.

Execution of standing limit orders by market orders is settled according to a set of priority or *precedence rules*. We employ the standard rule of order-driven markets, i.e. first price priority, then display priority and finally arrival time priority.

In general, at time of arrival t_0 and at the submission price level B_0, the trader faces a depth of D visible shares and H hidden shares that have higher time priority. We assume that a proportion c of shares of the visible depth cancel before the next market order arrives. Hence, right before market orders arrive at t_2, the order volume that has higher execution priority than the Iceberg orders displayed shares is

$$Q^d := D(1-c) + \hat{y}.$$

The uncancelled $D(1-c)$ shares and the number of $\hat{y}$ of newly incoming limit orders shares have price priority. At the same time,

$$Q^h := Q^d + \Delta + h + y$$

shares have priority over the Iceberg trader's hidden shares. This follows, because all shares Q^d that have priority over the Iceberg's displayed part do also have priority over its hidden part. Additionally, all the visible $\Delta + y$ shares have display priority, while the hidden depth h has time priority. Thus, the sequence of execution priority reads $(Q^d, h+y, N-\Delta)$ with the first entry representing the order volume of highest priority. The trader's total execution volume Z^Δ can then be represented by

$$Z^\Delta = \begin{cases} 0 & x \leq Q^d \\ x - Q^d & Q^d < x \leq \Delta + Q^d \\ \Delta & \Delta + Q^d < x \leq Q^h \\ \Delta + x - Q^h & Q^h < x \leq Q^h + N - \Delta \\ \Delta + (N-\Delta) & Q^h + N - \Delta < x. \end{cases} \tag{2.2.4}$$

The execution volume Z^Δ is given in terms of the observable quantities D , Δ, N, the cancellation ratio c and the random (unobservable) quantities h, y, $\hat{y}$, and x. Unobservable quantities are modelled as non-negative random variables. For notational purpose, we denote random variables by lower case letters and deterministic quantities by capital letters.

Order volumes on high-frequency time scales have been shown to possess a significant mass-of probability at zero, i.e. there is a significant probability that no orders arrive over short horizons (see Table 2.1). This is even more relevant for less liquid stocks. In particular, most simple continuous distribution (e.g., exponential) with positive support are misspecified on these time scales. Hence, Hautsch et al. (2013) propose general point-mass mixture distributions for trade volumes (i.e. liquidity demand). In our model, we extend the point-mass mixture approach to the full range of order flow, including the supply side of liquidity. Specifically and to keep the model as simple as possible, we propose a zero-augmented exponential distribution for the flow variables $x, y, \hat{y}$ and the hidden liquidity variable h, i.e. the respective densities are:

$$f_y(s) = (1-q)\cdot \mathbf{1}_{\{s=0\}} + \tfrac{q}{\beta}\cdot e^{-\frac{s}{\beta}}\cdot \mathbf{1}_{\{s>0\}} \tag{2.2.5}$$

$$f_{\hat{y}}(t) = (1-\hat{q})\cdot \mathbf{1}_{\{t=0\}} + \tfrac{\hat{q}}{\hat{\beta}}\cdot e^{-\frac{t}{\hat{\beta}}}\cdot \mathbf{1}_{\{t>0\}} \tag{2.2.6}$$

$$f_x(u) = (1-p)\cdot \mathbf{1}_{\{u=0\}} + \tfrac{p}{\alpha}\cdot e^{-\frac{u}{\alpha}}\cdot \mathbf{1}_{\{u>0\}} \tag{2.2.7}$$

$$f_h(v) = (1-r)\cdot \mathbf{1}_{\{v=0\}} + \tfrac{r}{\gamma}\cdot e^{-\frac{v}{\gamma}}\cdot \mathbf{1}_{\{v>0\}}\ , \tag{2.2.8}$$

where $\mathbf{1}$ denotes the indicator function. With our choice of density functions the expected transaction volume can be given in closed form. This renders our model amenable to some theoretical analysis. In particular, using the fact that y, $\hat{y}$, x and h are independent, the expected execution volume reads

$$V := E[Z^{\Delta}] = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} Z^{\Delta} f_y(s) f_{\hat{y}}(t) f_x(u) f_h(v)\, ds\,dt\,du\,dv. \tag{2.2.9}$$

The proof of the next proposition is provided in the appendix.

Proposition 1 (Expected Execution Volume)**.** *Assuming that the expected market order flow is positive and the order size to be traded is positive, i.e.* $p\cdot\alpha > 0$ *and* $N > 0$ *holds, then*

$$V := \alpha p (1-\hat{\beta}_r) e^{-\frac{D\,(1-c)}{\alpha}} \left\{ (1-\beta_r)(1-\gamma_r)\left(e^{-\frac{\Delta}{\alpha}} - e^{-\frac{N}{\alpha}}\right) + \left(1 - e^{-\frac{\Delta}{\alpha}}\right)\right\}, \tag{2.2.10}$$

with

$$\hat{\beta}_r := \hat{q}\frac{\hat{\beta}}{\alpha+\hat{\beta}}, \qquad \beta_r := q\frac{\beta}{\alpha+\beta}, \qquad \gamma_r := r\frac{\gamma}{\alpha+\gamma}. \tag{2.2.11}$$

In particular, the expected execution volume is bounded by the expected amount of arriving market order volume:

$$0 < E[Z^{\Delta}] \le p\cdot\alpha e^{-\frac{D}{\alpha}}\left(1 - e^{-\frac{N}{\alpha}}\right) < p\cdot\alpha. \tag{2.2.12}$$

The first term in the curly brackets in (2.2.10) corresponds to the execution of the hidden part of the Iceberg order. It depends on the parameters characterising submission-level liquidity (γ,β) relative to the market order volume (α), the total order size N, and the display ratio relative to the expected market order volume. The terms $(1-\beta_r)$ and $(1-\gamma_r)$ reflect the loss in time-priority the hidden part suffers due to incoming visible and standing hidden orders at the submission price level, respectively. The quantity $(1 - e^{-\frac{\Delta}{\alpha}})$ corresponds to the execution of the visible part; it only depends on Δ. The benchmark case $\Delta = N$ captures the limit order as the fully displayed case. In this case, only the second term matters.

In what follows, we implicitly assume that $\alpha \cdot p > 0$ and $N > 0$. To understand how exposure Δ affects the execution performance, it is necessary to understand how the trading cost $W = W(\Delta)$ is affected by changes in market parameters. A first idea derives from the partial derivatives of W using (2.2.2) and (2.2.10).

Lemma 1. *The partial derivatives of W obey*

$$\frac{\partial W}{\partial \Delta} < 0, \qquad \frac{\partial W}{\partial \alpha} < 0, \qquad \frac{\partial W}{\partial \hat{\beta}} > 0, \qquad \frac{\partial W}{\partial \beta} > 0, \qquad \frac{\partial W}{\partial \mu} > 0. \tag{2.2.13}$$

The last four inequalities are intuitive. If liquidity competition increases and prices move away, transaction costs must increase likewise. On the other hand, if more people are willing to trade against the trader, then his transaction costs should decrease. For a detailed proof, see Lemma 2 in the appendix.

Notice that the explicit dependency in Δ is capturing priority-gain that is associated with exposure and thus must be strictly negative, i.e. reducing transaction costs. However, that does not imply that exposure reduces overall transaction costs in general. Since exposure does also affect transaction costs implicitly through the order flows and prices, i.e. α, β, ..., these contributions may offset the priority-gains. Hence, in general the total derivative $\frac{dW}{d\Delta}$ will not be negative. We provide a more detailed analysis in the next sections.

2.2.2 Exposure Impact and the Order Imbalance

We accommodate the empirical fact that visible changes in the order imbalance affect market properties.[7] For instance, Esser and Mönch (2007) suggests that exposure affects the drift of the price process. To account for the microstructure of limit order markets, we extend this concept to incorporate order flows. More precisely, we assume that all free model parameters are functions of the imbalance I, i.e.

$$I \mapsto q_I, \hat{q}_I, p_I, \beta_I, \hat{\beta}_I, \alpha_I, r_I, \gamma_I, \mu_I, \tag{2.2.14}$$

where I denotes the *relative order book imbalance*,

$$I = I(\Delta) = \frac{D_{\,bid} - D_{\,ask} + \Delta}{D_{\,bid} + D_{\,ask} + \Delta}, \qquad \Delta \in [0, N], \tag{2.2.15}$$

where D_{bid} and $D_{\,ask}$ denote the visible standing volume at the best bid and ask respectively. Positive values represent bid-side liquidity excess, while negative represent sell-side excess. Thus, the trader's expected execution price (2.2.2) is a function of both an explicit and an implicit dependency on Δ:

$$W(\Delta, I(\Delta)) := W\Big(\Delta, \alpha(I(\Delta)), \beta(I(\Delta)), \hat{\beta}(I(\Delta)), \mu(I(\Delta))\Big), \tag{2.2.16}$$

[7]See for instance, Ranaldo (2004), Cao et al. (2009).

where for ease of notation and simplicity, we only consider dependency in α, β, $\hat{\beta}$ and μ but not in the order arrival probabilities $q, \hat{q}$ and p. We also assume that cancellations are not affected by the imbalance. The parameters of hidden depth at the submission price level r and γ don't depend on I as well, since the hidden orders have been submitted before the Iceberg trader arrived and can not be changed afterwards.

2.2.3 Analytical Discussion

By taking the total derivative of (2.2.16), using the chain rule and re-arranging terms, we can decompose the impact of infinitesimal changes in the display size into an explicit (priority-gain) and implicit (exposure impact) contribution, i.e.

$$\begin{aligned}\frac{d}{d\Delta}W &= I'(\Delta)\underbrace{\left(\frac{\partial\alpha}{\partial I}\frac{\partial}{\partial\alpha}+\frac{\partial\beta}{\partial I}\frac{\partial}{\partial\beta}+\frac{\partial\hat{\beta}}{\partial I}\frac{\partial}{\partial\hat{\beta}}+\frac{\partial\mu}{\partial I}\frac{\partial}{\partial\mu}\right)W}_{=:M_{Market}} - \underbrace{\left(-\frac{\partial W}{\partial\Delta}\right)}_{=:M_{Priority}>0} \\ &= I'(\Delta)M_{Market} - M_{Priority}.\end{aligned} \tag{2.2.17}$$

If the mapping $\Delta \mapsto W(\Delta)$ is strictly (quasi-) convex than the unique optimal display size $\Delta^* \in (0, N)$ is characterised by the first order condition $\frac{d}{d\Delta}W = 0$, or equivalently

$$I'(\Delta)M_{\text{Market}}(\Delta^*) = M_{\text{Priority}}(\Delta^*).$$

Thus, the optimal size marks the trade-off between priority-gains and losses due to market (exposure) impact. Notice that by Lemma 2, the term $-M_{Priority} = \frac{\partial W}{\partial\Delta}$ is strictly negative, while the market impact contribution M_{Market} depends on how flows and prices changes with the imbalance I, see (2.2.14). Due to the highly non-linear dependence on the display size, in general closed-form solutions for the optimal display size will not be available. Instead, we confine our theoretical analysis to certain market impact specifications and asymptotic cases.

Absence of Exposure Impact

In this section, we briefly discuss the implications for exposure strategies when exposure does not adversely affect markets. This is typically the case when markets are very liquid or the investor is economically small.

Corollary 1. *Assume that exposure does not adversely affect market properties, i.e. liquidity competition does not increase, liquidity demand does not decrease and prices do not move away when investors expose, i.e.,* $-\frac{\partial\alpha}{\partial I}, \frac{\partial\beta}{\partial I}, \frac{\partial\hat{\beta}}{\partial I}, \frac{\partial\mu}{\partial I} \leq 0$. *Then*

$$\Delta^* = N \tag{2.2.18}$$

Proof. One needs to show that (2.2.17) is strictly negative, i.e. $\frac{dW}{d\Delta} < 0$. Since according Lemma 1 and (2.2.15) $\frac{\partial W}{\partial\Delta} < 0$ and $I'(\Delta) > 0$ holds, it suffices to show that $M_{Priority} < 0$.

But this follows directly from the assumption and the signs of the partial derivatives of W (see Lemma 1). $\square$

In the preceding case, the absence of adverse exposure effects allows the trader to fully benefit from gains in time-priority. However in reality, these exposure-rewarding scenarios are not very likely. In fact, and in line with our findings in section 2.3, the literature mostly suggests that market properties are adversely affected by exposure: higher liquidity competition, i.e. $\frac{\partial \hat{\beta}}{\partial I} > 0$ (c.f. Harris (1997) and Buti and Rindi (2013)), lower liquidity demand $\frac{\partial \alpha}{\partial I} < 0$ (c.f. Moinas (2010)) and away-drifting prices, i.e. $\frac{\partial \mu}{\partial I} > 0$ (c.f. Hautsch and Huang (2012)). Consequently, in general traders are forced to hide at least a portion of their order. In the next section, we discuss how the presence of *parasitic traders* can lead to this outcome.

Parasitic Trading: Liquidity Competition vs Time Priority

Harris (1997) argues that "parasitic traders" have various motifs to front-run or "quote-match" large exposed orders, by selling and buying at more aggressive prices. In our framework, this mechanism is captured by the flow parameter $\hat{\beta}$. Assuming that all other parameters do not depend on Δ, the trade-off between exposure impact and priority loss is governed by (2.2.17) and reads

$$\frac{d}{d\Delta}W = I'(\Delta)\frac{\partial \hat{\beta}}{\partial I}\frac{\partial}{\partial \hat{\beta}}W - \left(-\frac{\partial W}{\partial \Delta}\right). \tag{2.2.19}$$

Assuming that exposure impact is linear in liquidity competition, i.e.

$$\hat{\beta}(\Delta) = \hat{\beta}_0 + \hat{\beta}_1\Delta, \tag{2.2.20}$$

we show in Proposition 2 below that the the optimal display size Δ^* can be uniquely characterised in terms of the real-valued negative branch of the Lambert function Φ_{-1} (c.f. Corless et al. (1996)).[8] A Lambert function Φ is any function that solves the equation:

$$w = \Phi(w)e^{\Phi(w)}, w \in \mathbb{C}. \tag{2.2.21}$$

The real-valued part of the negative branch Φ_{-1} is defined for the interval $[-1/e, 0]$. It is strictly negative, unbounded from below, monotonically decreasing and obeys $\Phi_{-1} \leq \Phi_{-1}(-1/e) = -1$.

Proposition 2. *We use the notation* $\eta := (1-\gamma_r)(1-\beta_r)$. *Assume* $N > 0$, $\alpha > 0$ *and* $\hat{\beta} = \hat{\beta}_0 + \hat{\beta}_1\Delta$ *with* $\hat{\beta}_0 \geq 0$, $\hat{\beta}_1 > 0$ *and* $\frac{\partial \alpha}{\partial \Delta} = \frac{\partial \beta}{\partial \Delta} = \frac{\partial \mu}{\partial \Delta} = 0$.

(i) If $e^{-\frac{\alpha+\hat{\beta}_0}{\alpha\hat{\beta}_1}} < \frac{1-\eta}{1-e^{-\frac{N}{\alpha}}\eta}$ *holds and using the notation* $\overline{w} = -e^{-1-\frac{\alpha+\hat{\beta}_0}{\hat{\beta}_1\alpha}}\frac{(1-e^{-\frac{N}{\alpha}}\eta)}{1-\eta}$, *the*

[8] We use here a different notation. The standard notation for the Lambert function is W and it is also called the Lambert W-function or the Product-Log function.

unique optimal display size Δ^* is given by

$$\Delta^* = \min\left[\max\left[-\frac{\alpha+\hat{\beta}_0}{\hat{\beta}_1} - \alpha(1+\Phi_{-1}(\overline{w})), 0\right], N\right]. \tag{2.2.22}$$

(ii) If $e^{-\frac{\alpha+\hat{\beta}_0}{\alpha\hat{\beta}_1}} \geq \frac{1-\eta}{1-e^{-\frac{N}{\alpha}}\eta}$, the unique optimal display size Δ^* is

$$\Delta^* = 0. \tag{2.2.23}$$

The intuition for this result is as follows: when liquidity competition at the submission price level is significant, i.e. η is low, condition (i) holds. Due to the increased competition, hiding orders implies a larger loss in time-priority. On the other hand, when liquidity competition at the submission price level is low, i.e. $\eta \approx 1$, then condition (ii) prevails. In this case, hiding is not costly at all as there is no competitor to lose time-priority against. Hence in this case, it is optimal to hide all shares.

Corollary 2. *The display size is*

- *decreasing with the trade size N,*
- *decreasing in the exposure impact $\hat{\beta}_1$,*
- *increasing in $\hat{\beta}_0$,*
- *increasing with submission level liquidity competition $1-\eta$.*

The monotonicity statements follow directly from proposition 2. For instance, large orders need to be kept hidden as the associated exposure impact materialises in higher costs. An important fact is that the decision to hide or display does depend on liquidity competition on all relevant price levels: not only at the submission price level, but also at better price levels.

Asymptotic Discussion

When the probability of execution is low, the trader has to trade an increasing portion of the order by market orders against the best ask price. In this case, costs related to exposure are mainly driven by price, not the order flow component. Execution probabilities can be low, when the order size is large, liquidity competition is high and liquidity demand is low. In this section, we address these cases as asymptotic limits.

We assume that the price impact is monotone in the exposure size, or equivalently it is monotone in the order imbalance I. In particular, we shall distinguish the two cases: (i) $\frac{\partial\mu}{\partial I} > 0$ (prices move away from the side of exposure) and (ii) $\frac{\partial\mu}{\partial I} < 0$ (prices convert to the side of exposure). Our first result shows that price impact is more important than order flow impact when orders are large.

Proposition 3. *Suppose that α, β, $\hat{\beta}$ and μ are Lipschitz continuous in I with constant K and that α and μ are bounded. Then there is a $N_d > 0$ such that for any order size with $N > N_d$, the optimal display size $\Delta^*(N)$ obeys*

$$\Delta^*(N) = \begin{cases} 0 & \frac{\partial}{\partial I}\mu > 0 \\ N & \frac{\partial}{\partial I}\mu < 0 \end{cases} . \tag{2.2.24}$$

Proof. First we show that $V = E[Z^\Delta]$ is Lipschitz-continuous in Δ. To see this, we take its total derivative

$$\begin{aligned} \left|\frac{dV}{d\Delta}\right| &= \left|I'(\Delta)\left(\frac{\partial\alpha}{\partial I}\frac{\partial}{\partial\alpha} + \frac{\partial\beta}{\partial I}\frac{\partial}{\partial\beta} + \frac{\partial\hat{\beta}}{\partial I}\frac{\partial}{\partial\hat{\beta}}\right)V + \frac{\partial V}{\partial\Delta}\right| \\ &\leq \frac{2D_{ask}}{(D_{ask}+D_{bid})^2}\left(\left|\frac{\partial\alpha}{\partial I}\right| + \left|\frac{\partial\beta}{\partial I}\right| + \left|\frac{\partial\hat{\beta}}{\partial I}\right|\right) + 1 \\ &\leq \frac{6KD_{ask}}{(D_{ask}+D_{bid})^2} + 1, \end{aligned} \tag{2.2.25}$$

where in the first inequality we used the fact that $|\frac{\partial V}{\partial s}| \leq 1$ for $s \in \left\{\alpha, \hat{\beta}, \beta\right\}$ according to Lemma 3 (see appendix). Hence using (2.2.2), we find

$$\begin{aligned} \lim_{N\to\infty} W'(\Delta) &= \lim_{N\to\infty}\left(-\frac{\mu(\Delta)}{N}\frac{dV}{d\Delta} + \left(1 - \frac{V}{N}\right)\frac{d\mu(\Delta)}{d\Delta}\right) \\ &= \underbrace{-\lim_{N\to\infty}\frac{1}{N}\left(\mu(\Delta)\frac{dV}{d\Delta}\right)}_{=0} + \frac{d\mu(\Delta)}{d\Delta} - \underbrace{\lim_{N\to\infty}\frac{V}{N}\frac{d\mu(\Delta)}{d\Delta}}_{=0}. \end{aligned} \tag{2.2.26}$$

In the limit, the first term vanishes uniformly in Δ, since μ and $\frac{\partial V}{\partial\Delta}$ are uniformly bounded in Δ. The third term vanishes, since V is bounded by α according to (2.2.12) and by assumption α is uniformly bounded and μ is uniformly Lipschitz. Finally, the result follows from

$$\frac{d}{d\Delta}\mu(I(\Delta)) = \frac{d\mu(I)}{dI}\underbrace{I'(\Delta)}_{>0}. \tag{2.2.27}$$

$\square$

In the next Proposition, we show that opportunity costs dominate when either liquidity demand is very low or liquidity competition is very strong.

Proposition 4 (Liquidity Flow and Optimal Display Size)**.** *For sufficiently high liquidity competition $\hat{\beta}$, and for sufficiently low liquidity demand α, the optimal display size obeys*

$$\Delta^* = \begin{cases} 0 & \frac{\partial\mu}{\partial I} > 0 \\ N & \frac{\partial\mu}{\partial I} < 0 \end{cases} . \tag{2.2.28}$$

Proof. According to Lemma 2, in the limit $\hat{\beta} \to \infty$ and $\alpha \to 0$, the partial derivatives obey

$$\frac{\partial W}{\partial \Delta} \to 0, \qquad \frac{\partial W}{\partial \beta} \to 0, \qquad \frac{\partial W}{\partial \hat{\beta}} \to 0. \tag{2.2.29}$$

More cumbersome, yet straightforward, it also follows that $\frac{\partial W}{\partial \alpha} \to 0$ holds. Hence overall, we have

$$\lim_{\alpha \to 0} W'(\Delta) = \lim_{\hat{\beta} \to \infty} W'(\Delta) = I'(\Delta) \frac{\partial \mu}{\partial I}, \quad \Delta \in [0, N] \tag{2.2.30}$$

The conclusion follows in the same way as in the proof to Proposition 3. □

According to (2.2.17), the structure of the market impact-term suggests that adding (i.e. exposing) only small orders to already large volumes at the top-of-the-book, or already large imbalances, will not alter the imbalance substantially. As we show in the next proposition, in this case the trader only weakly affects the market, while she fully benefits from the gain in time-priority. Ultimately, the trader is be better-off exposing his trading intentions.

Proposition 5. *For sufficiently large (negative) or small (positive) opposite-side depth D_{ask} (initial imbalance I_0), the optimal display size of a buy Iceberg trader obeys*

$$\Delta^* = N. \tag{2.2.31}$$

Proof. In view of (2.2.15), we have $\frac{2D_{ask}}{(D_{bid}+D_{ask}+\Delta)^2} \le \frac{2D_{ask}}{(D_{bid}+D_{ask})^2}$. Hence, in the limit of sufficiently large or small opposite-side depth D_{ask}, $I'(\Delta)$ approaches zero uniformly in Δ. Since M_{Market} and $M_{Priority}$ do not depend on D_{ask}, it follows from (2.2.17) that for $D_{ask} \to 0$ or $D_{ask} \to \infty$, the term $I'(\Delta) M_{Market}$ vanishes uniformly in Δ while at the same time $M_{Priority} < 0$. Thus, in the limit $\lim_{D_{ask} \to 0} \frac{\partial W}{\partial \Delta} = \lim_{D_{ask} \to \infty} \frac{\partial W}{\partial \Delta} = M_{Priority} = \frac{dW}{d\Delta} < 0$ for all $\Delta \in [0, N]$. □

Exposure Indifference

As stated, the execution performance depends on the exposure size in two ways: an explicit contribution that refers to priority losses and an implicit contribution over order flows and price dynamics. When liquidity competition at the trader's submission price level is absent, i.e. $\eta := (1 - \beta_r)(1 - \gamma_r) = 1$, then there is no priority loss associated with hiding the order and thus the the explicit (priority-related) dependency with respect to exposure vanishes. In this case, when liquidity supply and demand fulfil a *balancing condition*, the execution performance is indifferent with respect to the exposure. The proof of the next Proposition is given in the appendix.

Proposition 6. *Assume $\eta = 1$ and that the effective spread μ is constant. Assume that the liquidity supply $\hat{\beta}$ and demand α fulfil a balancing condition, i.e.*

$$\frac{1}{\alpha} \frac{d\alpha}{dI} = \frac{d\hat{\beta}}{dI} \frac{e^{\frac{N}{\alpha}} - 1}{(e^{\frac{N}{\alpha}} - 1)\left(D(1 + \frac{\hat{\beta}}{\alpha}) + \alpha + 2\hat{\beta}\right) - N(1 + \frac{\hat{\beta}}{\alpha})}. \tag{2.2.32}$$

Then, the execution performance W does not depend on Δ.

If (2.2.32) holds, exposure effects exactly cancel each other out and there is no impact on trading costs. For instance, potential adverse effects that are related to increased liquidity competition ($\hat{\beta}$) are perfectly offset by higher liquidity demand (α). In particular, the market order volume has to increase (decrease) as the limit order flow increases (decreases).[9]

2.3 Empirical Analysis

In this section, we estimate the exposure mappings (2.2.14) to calculate optimal display size estimates for a range of stocks.

2.3.1 Data

Our estimates are based on Message Level data from the Instinet (INET)[10] market for the period of January and February 2009. This dataset provides messages for every order entry, including modification, cancellation, submission and execution. The messages contain order identification number, time stamps, modification, submission, cancellation and execution size, as well as a flag marking the side of the book (buy or sell). This way, we were able to track every order until cancellation/execution and to re-construct the visible order book.

In order to estimate the dependence of the model parameters on imbalances we used a sample of *non-intersecting* Δt-periods during 9:30 and 15:30 hrs for which - for each realisation of the initial imbalance I- we record the cumulative flow volumes $(x_I, \hat{y}_I, y_I)$, standing hidden volume (h_I) as well as the effective spread at the respective terminal time, and constructed the Maximum-Likelihood-estimates (MLE) for the corresponding flow $(q_I, \hat{q}_I, p_I, \beta_I, \hat{\beta}_I, \alpha_I)$ and price (μ_I) parameters. We might occasionally omit the different notation for "true" parameter values and empirical estimates if there is no risk of confusion.

The INET dataset does not send messages for modification and cancellation of *hidden orders* which renders the reconstruction of the hidden volume h_I at a given price level incomplete. In order to obtain proper estimates for the hidden parameters (r_I, γ_I), we use a second dataset, the NASDAQ ModelView. At each price level, this data set provides full minute-by-minute snapshots of the market's *consolidated* visible and hidden depth for NASDAQ, NYSE and AMEX-listed stocks. This includes the following selection of liquid, high-tech S&P 500-stocks from the INET-exchange: Cisco Systems Inc. (CSCO), Dell Inc. (DELL), eBay Inc. (EBAY), Hewlett-Packard Company (HPQ), Microsoft Corp. (MSFT) and Oracle Corp. (ORCL).

---

[9]This follows from the fact that the denominator of the right-hand side of (2.2.32) is strictly positive, since $(e^{\frac{N}{\alpha}} - 1)(\alpha + \hat{\beta}) \geq N(1 + \frac{\hat{\beta}}{\alpha})$ for $\alpha > 0$ and $\hat{\beta} > 0$.

[10]As of the last quarter of 2008 INET holds 5% share of the total US market in traded equity volume.

Descriptive Statistics

Table 2.2 and 2.1 report unconditional time averages for our model parameters. We want to highlight some important microstructure effects. First, cancellation of standing orders is substantial. For the range of stocks selected, we observe that for most of the stocks, more than 50% of standing orders at the best prices do not get executed, but cancelled. A potential reason why orders do not get executed but cancelled frequently, is related to liquidity competition and price improvements. A lot of trade demand executes against orders that improved the prevailing price. Hence, standing orders face the risk of being undercut by more aggressive orders.

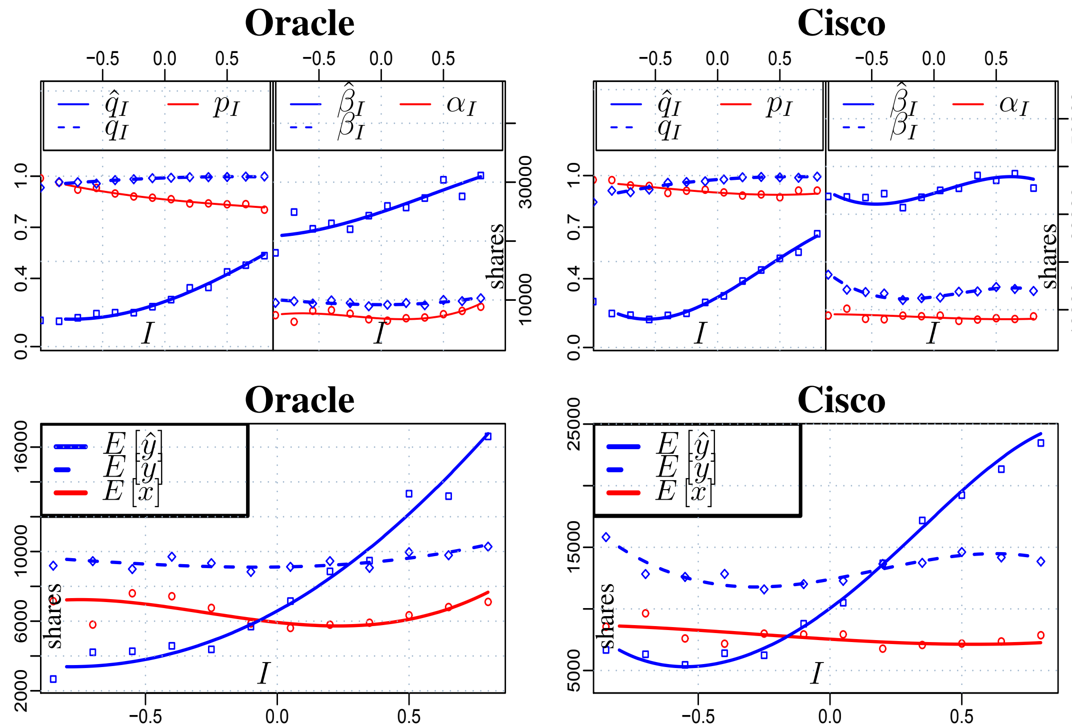
Table 2.3 reports correlation estimates between the expectations of cumulated incoming order flows and total price increment (returns) on a time horizon Δt and the (initial) order imbalance. The results can be summarised as follows. First, larger depth on one side (imbalance) increases also higher same-side liquidity competition, higher same-side liquidity demand and creates an opposite price-pressure. Second, the effect is most strongly for liquidity competition at better prices ($\hat{y}_I$) and the price return (μ_I). Correlation with respect to liquidity competition at the best price level (y) as well as liquidity demand (x) is less significant. Third, the correlation between initial imbalance and market properties is most strongest for short time horizons (i.e. 3 to 10 seconds) and decreases for longer horizons (30 seconds).

2.3.2 Market Impact Estimates

For the estimation purpose, we assume a discretisation of the imbalance interval of 0.15 points and restrict our analysis to a range between -0.7 and 0.7 as imbalances beyond these regions don't gather sufficient statistics. For each realisation of *I*, we construct the respective Maximum Likelihood Estimates (MLE) for the model parameters $q_I, \hat{q}_I, p_I, \beta_I, \hat{\beta}_I, \alpha_I, r_I, \gamma_I, \mu_I$. In order to obtain smooth functional representations, we additionally apply a *cubic weighted Ordinary Least Squares* (wOLS) on the point estimates. This is a proper compromise between simplicity and the desire to cover nonlinear, asymmetric effects as well as heteroscedacity. For $\Delta t = 30s$, examples of estimated conditional probabilities ($q_I, \hat{q}_I, p_I$), conditional mean volumes ($\beta_I, \hat{\beta}_I, \alpha_I$) and the terminal best ask price A_T are shown in Figure 2.1 and 2.2 for the case of ORCL and CSCO. Results for the other stocks are provided in the appendix, see figures 2.3 and 2.4.

The estimation results can be summarised as follows. First, confirming the correlation analysis, the impact of changes in the visible imbalance on liquidity competition at aggressive prices ($\hat{y}$) and the price return is most significant. For instance, for Oracle, liquidity competition on the buy side increases almost 5 fold from extreme sell-side to the extreme buy-side imbalances. On the other hand, changes in the imbalance affect liquidity demand (x) and liquidity competition at same price levels (y) only weakly. Second, the imbalance impact on order flows is felt most strongly for short time horizons. The imbalance impact on $\hat{y}$ is only half as strong over a 30-seconds time period as compared to a 3-seconds time period. Third, the imbalance impact on returns is most strongly for longer time horizons.

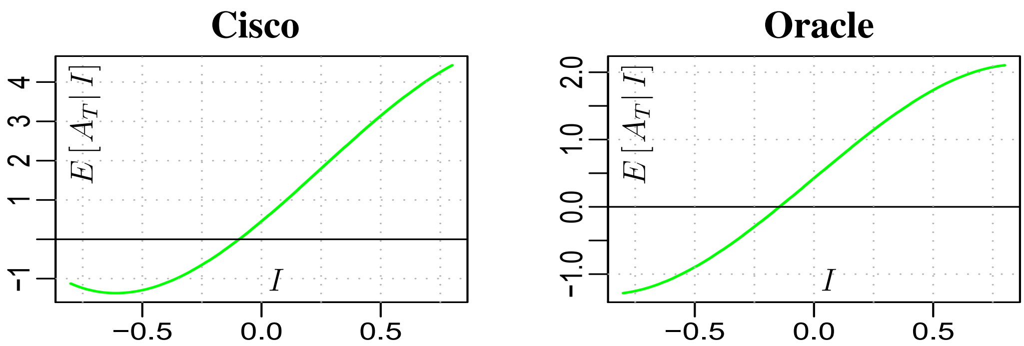
Figure 2.1: Example of flow parameter estimates for two random stocks (Oracle and Cisco) as a function of imbalance for $\Delta t = 30s$. Estimation is based on weighted OLS of cubic polynomials. $\hat{q}$, q and p refer to the probabilities of the different limit order ($\hat{y}$ and y) arrivals and market order arrivals (x). Similarly, $\hat{\beta}$, β and α refer to their expected order arrival sizes. The unconditional expected order volumes are given in the figure below. Notice that $E[\hat{y}] = \hat{q}\hat{\beta}$, $E[x] = p\alpha$ and $E[y] = q\beta$ holds.



2.3.3 Optimal Exposure Estimates

Using the estimates from (2.2.14) and the transaction cost model (2.2.12), we can numerically derive the optimal display choice (Δ^*) according to (2.2.3) and analyse its dependence on different settings and market environments. In order to facilitate a better comparison, we introduce the optimal display ratio $\Delta_r^* := \frac{\Delta^*}{N}$. Figures 2.5 and 2.6 show the results for the case $\Delta t = 10s$. The trader's optimal display ratios are drawn with respect to the initial buy-side depth (D_{bid}) and initial imbalance (I_0) he observes at arrival time t_0. That is, at arrival time our trader observes same side liquidity of D_{bid} and a total (relative) order imbalance of I_0. Optimal exposure strategies close to 1 (i.e. full exposure) are coloured red; exposure strategies close to 0 (i.e. zero exposure) are coloured blue. Optimal exposure strategies for intermediate exposures are coloured using a specific rainbow-colour-gradient. The initial same-side depth (D_{bid}) ranges have been chosen so as to include typical average best bid depths for the respective stocks; see Table 2.2 in the Appendix. For order sizes (N), we have chosen three values for each stock: a small, an intermediate and a large one.

Figure 2.2: Example of price parameter estimates for two random stocks (Oracle and Cisco) as a function of imbalance for $\Delta t = 30s$. We report, the best ask price $E\left[A_T | I\right]$ relative to the initial order book imbalance I with time period $\Delta t = 30s$. Estimation is based on weighted OLS of using cubic polynomials.



The results can be condensed into three points. First, throughout the set of stocks, for small order sizes, total exposure is optimal, i.e. $\Delta_r^* = 1$. Exceptions occur for some stocks for large initial same-side depth (D_{bid}), as in the case of Cisco, Dell, Ebay and Oracle. However, as our markets obey $\frac{\partial \mu}{\partial I} > 0$ (See figure 2.4), this is consistent with Proposition 5. Second, for large order sizes zero-exposure (i.e. hiding the order) is the best strategy. Again taking into account that our markets obey $\frac{\partial \mu}{\partial I} > 0$, this is in line with Proposition 2. Intermediate order sizes generally lead to medium exposure. However, here the optimal strategy depends critically on the initial depth and imbalance at arrival-time. That is, exposure is optimal for *mid-size* orders, when there is opposite-side liquidity-excess (negative imbalance).

The results are easy to interpret. Large traders need to hide their orders as showing the full order would substantially change the open and displayed order book, in particular the order imbalance. However, changes in the order imbalances affect incoming order flows such that exposure adversely affects the limit orders transaction process. One main reason, as elaborated in the previous sections, is the fact that liquidity competition increases at the expense of the limit order trader. Small orders on the other hand, as they don't significantly alter the state of the book, do not cause substantial adverse affects (market impact) and thus can be safely shown to the market. For mid-size orders, traders need to take additional market dimensions into account, such as the number of shares having higher priority D_{bid} and the prevailing imbalance I_0. These observations are also in line with empirical findings.[11]

2.3.4 The Benchmark Test: Stealth versus Sunshine Trading

To quantify the advantage of hiding over exposing trade interests, let denote $W_{ice} := \min_{0 \le \Delta \le N} W(\Delta)$ the transaction costs of the Iceberg order and $W_{lim} := W(N)$ the transaction costs of the ex-

[11] See Bessembinder et al. (2009).

posed limit order. We consider the relative difference between both performance measures

$$\sigma := \frac{W_{ice} - W_{lim}}{W_{lim}} \leq 0. \tag{2.3.1}$$

By construction, the smaller σ, the higher the benefit of hiding trade interests and using Iceberg orders. If σ is close to zero, there is no additional significant benefit. We compute σ with respect to different market settings. The results are shown for Apple, Amazon, Cisco, Dell, Ebay, Hewlett-Packard, Microsoft and Oracle for $\Delta t = 10s$ in figures 2.8 and 2.7. Figure 2.7, shows σ as a function of the (initial) depth D_{bid} and the book-imbalance I_0; Figure 2.8 plots the performance depending on order sizes N and imbalances I_0. Green-blue-coloured (red-coloured) faces represent regions, where Icebergs do (not) provide additional cost-savings as compared to plain limit orders.

The results are robust among stocks and are as follows. First, adopting a stealth strategy is most beneficial when the order imbalance is initially skewed towards the trader's side of the market and the trader's order size is large. In these cases, the relative advantage of using an Iceberg order can be significant, performing up to 60% better than exposed limit orders. Third, Iceberg orders are least effective when the initial order imbalance is skewed towards to opposite side of the market and the order size is small.

2.4 Conclusion

We propose a structural model of Iceberg order execution in a limit order book market. Exposure is associated with costs but also benefits. The trader has to find the right balance, the optimal display size. In our model, the exposure benefits are associated with priority gain, as the standard policy in order-driven markets give displayed orders higher priority over hidden orders. However, the downside is exposure impact. Market dynamics might be adversely impacted by the trader's decision to expose his trading interests. In particular, the presence of parasitic or predatory traders forces the hidden trader to hide at least a portion of his order. These traders use the information revealed to them to undercut the limit order trader at his expense.

Our framework explicitly captures the trade-off between exposure benefits and costs and allows to derive several analytical predictions with respect to the motifs of hidden order submission. In general, traders hide more when their orders are large, markets react stronger to their exposed orders and when liquidity competition at the submission price level is low. In situations when the probability to execution is sufficiently low, only the price dimension of exposure impact affects the trader's trading performance. Order flows do not play a role. This is relevant when the order size to be traded is very large, liquidity competition is very high and liquidity demand is very low.

Based on high-frequency order book data, we provide empirical estimates on optimal display sizes for a range of stocks and under varying states of the market. Comparing the performance of Iceberg versus plain limit orders, we find that Iceberg orders can significantly boost trading performance, particularly when the initial order imbalance is sufficiently skewed towards the trader's side and/or the size to be traded is large.

Appendix 2.A Proofs

Proof of Proposition 1. Using the law of total expectation for partitions of the state space, we can write the expected execution volume $V := E[Z^{\Delta}]$ as a combination of the conditional expectations in the following sense:

$$V == \sum_{i=1}^{8} V_i,$$

with

$$
\begin{aligned}
V_1 &:= P\Big[x \neq 0, \hat{y} = 0, y = 0, h = 0\Big] E\Big[Z^{\Delta} | x \neq 0, \hat{y} = 0, y = 0, h = 0\Big] \\
V_2 &:= P\Big[x \neq 0, \hat{y} = 0, y \neq 0, h = 0\Big] E\Big[Z^{\Delta} | x \neq 0, \hat{y} = 0, y \neq 0, h = 0\Big] \\
V_3 &:= P\Big[x \neq 0, \hat{y} = 0, y = 0, h \neq 0\Big] E\Big[Z^{\Delta} | x \neq 0, \hat{y} = 0, y = 0, h \neq 0\Big] \\
V_4 &:= P\Big[x \neq 0, \hat{y} = 0, y \neq 0, h \neq 0\Big] E\Big[Z^{\Delta} | x \neq 0, \hat{y} = 0, y \neq 0, h \neq 0\Big] \\
V_5 &:= P\Big[x \neq 0, \hat{y} \neq 0, y \neq 0, h \neq 0\Big] E\Big[Z^{\Delta} | x \neq 0, \hat{y} \neq 0, y \neq 0, h \neq 0\Big] \\
V_6 &:= P\Big[x \neq 0, \hat{y} \neq 0, y \neq 0, h = 0\Big] E\Big[Z^{\Delta} | x \neq 0, \hat{y} \neq 0, y \neq 0, h = 0\Big] \\
V_7 &:= P\Big[x \neq 0, \hat{y} \neq 0, y = 0, h \neq 0\Big] E\Big[Z^{\Delta} | x \neq 0, \hat{y} \neq 0, y = 0, h \neq 0\Big] \\
V_8 &:= P\Big[x \neq 0, \hat{y} \neq 0, y = 0, h = 0\Big] E\Big[Z^{\Delta} | x \neq 0, \hat{y} \neq 0, y = 0, h = 0\Big]
\end{aligned}
$$

and P referring to the probability mass. Notice that as of (2.2.4) the cases $x = 0$ would lead to zero execution volumes Z^{Δ}. Hence, for the expected execution volume, it is sufficient to consider only the states with $x \neq 0$. To compute each term, we use the fact that h, x, $\hat{y}$ and y are independent and obey the point-mixture distribution as of (2.2.8). Thus, the first term V_1 reads

$$
\begin{aligned}
V_1 &= P\Big[x \neq 0, \hat{y} = 0, y = 0, h = 0\Big] E\Big[Z^{\Delta} | x \neq 0, \hat{y} = 0, y = 0, h = 0\Big] \\
&= p(1-q)(1-\hat{q})(1-r) \int_0^{\infty} \tfrac{1}{\alpha} e^{-\frac{x}{\alpha}} Z^{\Delta}(x, y = 0, \hat{y} = 0, h = 0) dx \\
&= \alpha p(1-q)(1-\hat{q})(1-r) e^{-\frac{D}{\alpha}} \Big(1 - e^{-\frac{N}{\alpha}}\Big).
\end{aligned}
$$

The last term V_8 reads

$$
\begin{aligned}
V_1 &= P\Big[x \neq 0, \hat{y} \neq 0, y = 0, h = 0\Big] E\Big[Z^{\Delta} | x \neq 0, \hat{y} \neq 0, y = 0, h = 0\Big] \\
&= p\hat{q}(1-q)(1-r) \int_0^{\infty} \int_0^{\infty} \tfrac{1}{\alpha} \tfrac{1}{\hat{\beta}} e^{-\frac{x}{\alpha}} e^{-\frac{\hat{y}}{\hat{\beta}}} Z^{\Delta}(x, \hat{y}, y = 0, h = 0) dx d\hat{y} \\
&= \alpha^2 \frac{p\hat{q}(1-q)(1-r)}{(\alpha+\hat{\beta})(\alpha+\gamma)} e^{-\frac{D}{\alpha}} \Big(1 - e^{-\frac{N}{\alpha}}\Big).
\end{aligned}
$$

For brevity, we do not show the details of the other terms. The remaining proofs are straightforward and follow by analogy. □

Lemma 2 (Partial Derivatives). *Using $\eta = (1-\beta_r)(1-\gamma_r)$, the partial derivatives of W obey*

$$
\frac{\partial W}{\partial \Delta} = -\frac{\alpha p(1-\hat{\beta}_r) e^{-\frac{D\,(1-c)}{\alpha}}}{\alpha N} e^{-\frac{\Delta}{\alpha}} \Big\{1 - \eta\Big\} < 0, \tag{2.A.1}
$$

$$
\frac{\partial W}{\partial \hat{\beta}} = \frac{\alpha p e^{-\frac{D\,(1-c)}{\alpha}}}{N} \Big\{\eta\Big(e^{-\frac{\Delta}{\alpha}} - e^{-\frac{N}{\alpha}}\Big) + \Big(1 - e^{-\frac{\Delta}{\alpha}}\Big)\Big\} \frac{\hat{q}\hat{\beta}}{(\alpha+\hat{\beta})^2} > 0, \tag{2.A.2}
$$

$$
\frac{\partial W}{\partial \beta} = \frac{\alpha p(1-\hat{\beta}_r) e^{-\frac{D\,(1-c)}{\alpha}}}{N} \Big\{(1-\gamma_r)\Big(e^{-\frac{\Delta}{\alpha}} - e^{-\frac{N}{\alpha}}\Big)\Big\} \frac{q\beta}{(\alpha+\beta)^2} > 0, \tag{2.A.3}
$$

$$
\frac{\partial W}{\partial \alpha} < 0. \tag{2.A.4}
$$

Proof. The partial derivatives with respect to Δ, β and $\hat{\beta}$ follow from the definition of W in (2.2.2) and (2.2.10). The respective signs also follow directly from $\eta < 1$ and the fact that $N \geq \Delta$.

For the last inequality, note that that η and $\alpha e^{-\frac{D\,(1-c)}{\alpha}}$ are increasing for $D(1-c) \geq 0$. The function $(e^{-\frac{\Delta}{\alpha}} - e^{-\frac{N}{\alpha}})$ is increasing for $N \geq \Delta$ and $(e^0 - e^{-\frac{\Delta}{\alpha}})$ as well. Hence, the expected execution volume (2.2.10) is a product and sum of increasing functions in α, so it is increasing in α. Consequently, via (2.2.2) the expected transaction costs W is a decreasing function of α. □

Lemma 3 (Derivative bounds)**.** *The partial derivatives of the expected execution volumes are uniformly bounded.*

$$\left|\frac{\partial E[Z^\Delta]}{\partial \Delta}\right| \leq 1, \qquad \left|\frac{\partial E[Z^\Delta]}{\partial \hat{\beta}}\right| \leq 1, \qquad \left|\frac{\partial E[Z^\Delta]}{\partial \beta}\right| \leq 1, \qquad \left|\frac{\partial E[Z^\Delta]}{\partial \alpha}\right| \leq 1.$$

Proof. By (2.2.2), $\frac{\partial E[Z^\Delta]}{\partial s} = -N\frac{\partial W}{\partial s}$ for any s. Thus $\left|\frac{\partial E[Z^\Delta]}{\partial s}\right| = N\left|\frac{\partial W}{\partial s}\right|$. The rest follows from Lemma 3. For brevity, we do not provide a detailed proof for the partial derivative with respect to α. Its derivation is similar. □

Lemma 4 (First Derivative)**.** *Given the specifications of Proposition 2, i.e.* $\hat{\beta}(\Delta) = \hat{\beta}_0 + \hat{\beta}_1\Delta$ *with* $b > 0$ *and* $\frac{\partial \alpha}{\partial \Delta} = \frac{\partial \beta}{\partial \Delta} = \frac{\partial \mu}{\partial \Delta} = 0$*,* W *obeys*

$$\begin{aligned}\frac{dW}{d\Delta} = \frac{\mu\alpha p}{N}\frac{e^{-\frac{D(1-c)}{\alpha}}}{(\alpha+\hat{\beta}_1\Delta+\hat{\beta}_0)^2}\Bigg\{&\hat{\beta}_1\alpha\left(1-\eta e^{-\frac{N}{\alpha}}\right)\\ &+e^{-\frac{\Delta}{\alpha}}(\eta-1)\left(\alpha(1+\hat{\beta}_1)+\hat{\beta}_1\Delta+\hat{\beta}_0\right)\Bigg\},\\ \frac{d^2W}{d\Delta^2} = -\frac{p\mu}{N}\frac{e^{-\frac{D+N+\Delta}{\alpha}}}{(\alpha+\beta_1\Delta+\beta_0)^3}\Bigg\{&2\beta_1^2\alpha^2 e^{\frac{\Delta}{\alpha}}\left(e^{\frac{N}{\alpha}}-\eta\right)\\ +e^{\frac{N}{\alpha}}(\eta-1)\Big((\beta_0+\beta_1\Delta)^2&+\alpha(2(1+\beta_1)(\beta_1(\alpha+\Delta)+\beta_0)+\alpha)\Big)\Bigg\}.\end{aligned}$$

Proof. For the sake of brevity, we confine our proof to the first derivative. The second derivative can be derived from the first derivative accordingly. Since $\hat{\beta}(\Delta) = \hat{\beta}_0 + \hat{\beta}_1\Delta$, $\eta = (1-\beta_r)(1-\gamma_r)$, using (2.2.2) and (2.2.10), we find

$$W = \frac{\mu}{N}\left(N - \alpha p(1-\hat{\beta}_r(\Delta))e^{-\frac{D(1-c)}{\alpha}}\left(\eta(e^{-\frac{\Delta}{\alpha}} - e^{-\frac{N}{\alpha}}) + 1 - e^{-\frac{\Delta}{\alpha}}\right)\right).$$

In the next step, we take the derivative using the fact that

$$\hat{\beta}_r'(\Delta) = \left(\tfrac{\hat{\beta}(\Delta)}{\alpha+\hat{\beta}(\Delta)}\right)' = \tfrac{\alpha\hat{\beta}'(\Delta)}{(\alpha+\hat{\beta}(\Delta))^2},$$

where prime denotes the derivative with respect to Δ.

$$
\begin{aligned}
\frac{dW}{d\Delta} &= \frac{\mu\alpha p}{N} e^{-\frac{D(1-c)}{\alpha}} \Bigg\{ \hat{\beta}_r'(\Delta)\Big(\eta(e^{-\frac{\Delta}{\alpha}} - e^{-\frac{N}{\alpha}}) + 1 - e^{-\frac{\Delta}{\alpha}}\Big) \\
&\qquad\qquad + \frac{1}{\alpha}(1-\hat{\beta}_r(\Delta))e^{-\frac{\Delta}{\alpha}}\Big(\eta - 1\Big)\Bigg\}, \\
&= \frac{\mu\alpha p}{N} e^{-\frac{D(1-c)}{\alpha}} \Bigg\{ \hat{\beta}_r'(\Delta)(1-\eta e^{-\frac{N}{\alpha}}) + e^{-\frac{\Delta}{\alpha}}(\eta-1)\Big(\hat{\beta}_r'(\Delta) + \frac{1}{\alpha}(1-\hat{\beta}_r(\Delta))\Big)\Bigg\}, \\
&= \frac{\mu\alpha p}{N} \frac{e^{-\frac{D(1-c)}{\alpha}}}{(\alpha+\hat{\beta}(\Delta))^2} \Bigg\{ \alpha\hat{\beta}'(\Delta)(1-\eta e^{-\frac{N}{\alpha}}) \\
&\qquad\qquad + e^{-\frac{\Delta}{\alpha}}(\eta-1)\Big(\alpha\hat{\beta}'(\Delta) + \alpha + \hat{\beta}(\Delta)\Big)\Bigg\}.
\end{aligned}
$$

From the last equation it is easy to see that the first assertion of the lemma holds by using the fact $\hat{\beta}(\Delta) = \hat{\beta}_0 + \hat{\beta}_1\Delta$. The second derivative follows straightforward and in a similar manner. $\square$

Proof of Proposition 2. Using Lemma 4, the first order condition reads

$$0 \overset{!}{=} e^{\frac{\Delta}{\alpha}} + B_1\Delta + B_0, \tag{2.A.5}$$

with $B_1 = \frac{\eta-1}{\alpha(1-\eta e^{-\frac{N}{\alpha}})}$ and $B_0 = \frac{(\eta-1)(\alpha(1+\hat{\beta}_1)+\hat{\beta}_0)}{\hat{\beta}_1\alpha(1-\eta e^{-\frac{N}{\alpha}})}$. By direct verification, one checks that the solution Δ_k^* to (2.A.5) can be expressed in terms of the Lambert functions Φ_k, as $\Delta_k^* = -\frac{1}{B_1}\left(B_0 + B_1\alpha\Phi_k\left(\frac{e^{-\frac{B_0}{B_1\alpha}}}{B_1\alpha}\right)\right)$. The Lambert functions solve the equation $w = e^{\Phi_k}\Phi_k$ for any $w \in \mathbb{C}$ for $k \in \mathbb{Z}$ (c.f. Corless et al. (1996)). Using the identities for B_0 and B_1, the solutions simplify to

$$\Delta_k^* = -\frac{\alpha+\hat{\beta}_0}{\hat{\beta}_1} - \alpha\Big(1+\Phi_k(w)\Big), \quad \text{with} \quad w = -e^{-1-\frac{\alpha+\hat{\beta}_0}{\hat{\beta}_1\alpha}}\frac{(1-e^{-\frac{N}{\alpha}}\eta)}{1-\eta}. \tag{2.A.6}$$

Notice that by constructions $w < 0$ and together with the first condition in the proof 2 $e^{-\frac{\alpha+\hat{\beta}_0}{\hat{\beta}_1\alpha}} < \frac{1-\eta}{1-e^{-\frac{N}{\alpha}}\eta}$ altogether imply $w \in [-1/e, 0)$. On the interval $[-1/e, 0)$, there are only two feasible real-valued branches of the Lambert functions Φ_0 and Φ_{-1} (c.f. Corless et al. (1996)). Since the principal branch obeys $\Phi_0(w) \geq -1$ for $w \in [-1/e, 0)$, by (2.A.6) it would give rise to strictly negative display sizes. Thus, the only and unique feasible solution is associated with the negative branch, i.e.

$$\Delta_{-1}^* = -\frac{\alpha+\hat{\beta}_0}{\hat{\beta}_1} - \alpha\Big(1+\Phi_{-1}(w)\Big). \tag{2.A.7}$$

It remains to be showns that this solution minimizes W. Therefore, using Lemma 4 and (2.A.7), we find

$$\frac{d^2W}{d\Delta^2}(\Delta^*_{-1}) = \underbrace{\frac{p\mu}{N}\frac{e^{-\frac{D(1-c)}{\alpha}}}{(\alpha+b\Delta^*_{-1}+\hat{\beta}_0)^3}}_{=:A>0}\Bigg\{2\hat{\beta}_1^2\alpha^2\left(\eta e^{-\frac{N}{\alpha}}-1\right)+e^{-\frac{\Delta^*_{-1}}{\alpha}}(1-\eta)\Bigg(\alpha^2$$

$$+\hat{\beta}_0^2+2(1+\hat{\beta}_1)\alpha(\alpha\hat{\beta}_1+\hat{\beta}_0)+2\hat{\beta}_1(\hat{\beta}_0+(1+\hat{\beta}_1)\alpha)\Delta^*_{-1}+\hat{\beta}_1^2(\Delta^*_{-1})^2\Bigg)\Bigg\}$$

$$= A\Bigg\{2\hat{\beta}_1^2\alpha^2\left(\eta e^{-\frac{N}{\alpha}}-1\right)$$

$$+e^{\frac{\alpha+\hat{\beta}_0}{\alpha\hat{\beta}_1}+1+\Phi_{-1}}(1-\eta)\Bigg(\alpha^2+\hat{\beta}_0^2+2(1+\hat{\beta}_1)\alpha(\alpha\hat{\beta}_1+\hat{\beta}_0)$$

$$+2\hat{\beta}(\hat{\beta}_0+(1+\hat{\beta}_1)\alpha)\Big(-\frac{\alpha+\hat{\beta}_0}{\hat{\beta}_1}-\alpha(1+\Phi_{-1})\Big)$$

$$+\hat{\beta}_1^2\Big(-\frac{\alpha+\hat{\beta}_0}{\hat{\beta}_1}-\alpha(1+\Phi_{-1})\Big)^2\Bigg)\Bigg\}.$$

Collecting terms with respect to $(1+\Phi_{-1})$ and using that $w = e^{-\frac{\alpha+\hat{\beta}_0}{\alpha\hat{\beta}_1}-1}\frac{\left(\eta e^{-\frac{N}{\alpha}}-1\right)}{1-\eta}$, yields

$$\frac{d^2W}{d\Delta^2}(\Delta^*_{-1}) = Ae^{\frac{\alpha+\hat{\beta}_0}{\alpha\hat{\beta}_1}+1+\Phi_{-1}}(1-\eta)\Bigg\{\underbrace{2\hat{\beta}_1^2\alpha^2e^{-\frac{\alpha+\hat{\beta}_0}{\alpha\hat{\beta}_1}-1}\frac{\left(\eta e^{-\frac{N}{\alpha}}-1\right)}{1-\eta}}_{=2w}e^{-\Phi_{-1}}$$

$$+\underbrace{2\alpha^2+2\hat{\beta}_0^2+2(1+\hat{\beta}_1)\alpha(\alpha\hat{\beta}_1+\hat{\beta}_0)+2\alpha\hat{\beta}_0-2(\hat{\beta}_0+(1+\hat{\beta}_1)\alpha)\Big(\alpha+\hat{\beta}_0\Big)}_{=2\alpha^2\hat{\beta}_1^2}$$

$$-2\hat{\beta}_1^2\alpha^2(1+\Phi_{-1})$$

$$+\hat{\beta}_1^2\alpha^2(1+\Phi_{-1})^2\Bigg\}$$

$$= \underbrace{A\hat{\beta}_1^2\alpha^2e^{\frac{\alpha+\hat{\beta}_0}{\alpha\hat{\beta}_1}+1+\Phi_{-1}}(1-\eta)}_{=:B>0}\Bigg\{2we^{-\Phi_{-1}}+\Big(1+\Phi_{-1}^2\Big)\Bigg\}.$$

Finally, using the Lambert identity $we^{-\Phi_{-1}(w)} = \Phi_{-1}(w)$ and $\Phi_{-1} = \Phi_{-1}(w)$, we obtain

$$\frac{d^2W}{d\Delta^2}(\Delta^*_{-1}) = B\left\{2\Phi_{-1}(w) + \left(1 + \Phi_{-1}(w)^2\right)\right\}$$
$$= B\left\{1 + \Phi_{-1}(w)\right\}^2 > 0.$$

The strict inequality follows from the fact that the negative branch obeys $\Phi_{-1} < -1$ for $w > -1/e$ (c.f. Corless et al. (1996)).

We still have to show the optimal display solution for the opposite case $e^{-\frac{\alpha+\hat{\beta}_0}{\alpha\hat{\beta}_1}} \geq \frac{1-\eta}{1-e^{-\frac{N}{\alpha}}}$. Therefore, recall the first order condition from Lemma 4

$$\frac{dW}{d\Delta} = \underbrace{\frac{\mu\alpha p}{N}\frac{e^{-\frac{D(1-c)}{\alpha}}}{(\alpha+\hat{\beta}_1\Delta+\hat{\beta}_0)^2}}_{=:A>0}\Big\{\hat{\beta}_1\alpha\left(1-\eta e^{-\frac{N}{\alpha}}\right)$$
$$+ e^{-\frac{\Delta}{\alpha}}(\eta-1)\left(\alpha(1+\hat{\beta}_1)+\hat{\beta}_1\Delta+\hat{\beta}_0\right)\Big\}.$$

Using the Taylor inequality $e^x \geq 1 + x$ for $x \geq 0$, yields

$$\frac{dW}{d\Delta} = \underbrace{A\hat{\beta}\alpha(1-\eta)e^{\frac{-\Delta}{\alpha}}}_{=:\hat{A}}\left(e^{\frac{\Delta}{\alpha}}\frac{1-\eta e^{-\frac{N}{\alpha}}}{1-\eta} - \frac{\alpha+\hat{\beta}_0}{\alpha\hat{\beta}_1} - 1 - \frac{\Delta}{\alpha}\right)$$
$$\geq \hat{A}\left(\left(1+\frac{\Delta}{\alpha}\right)\frac{1-\eta e^{-\frac{N}{\alpha}}}{1-\eta} - \left(\frac{\alpha+\hat{\beta}_0}{\alpha\hat{\beta}_1}+1\right) - \frac{\Delta}{\alpha}\right)$$
$$\geq \hat{A}\left(\left(1+\frac{\Delta}{\alpha}\right)\frac{1-\eta e^{-\frac{N}{\alpha}}}{1-\eta} - e^{\frac{\alpha+\hat{\beta}_0}{\alpha\hat{\beta}_1}} - \frac{\Delta}{\alpha}\right)$$
$$\geq \hat{A}\Bigg(\underbrace{\frac{1-\eta e^{-\frac{N}{\alpha}}}{1-\eta} - e^{\frac{\alpha+\hat{\beta}_0}{\alpha\hat{\beta}_1}}}_{\geq 0} + \frac{\Delta}{\alpha}\underbrace{\left(\frac{1-\eta e^{-\frac{N}{\alpha}}}{1-\eta} - 1\right)}_{>0}\Bigg) > 0.$$

□

Proof of Corollary 2. As of Proposition 2, the optimal display size has the form

$$\Delta^* = \min\Big[\max[\chi, 0], N\Big].$$

with

$$\chi = -\frac{\alpha + \hat{\beta}_0}{\hat{\beta}_1} - \alpha(1 + \Phi_{-1}(w)), \qquad w = -e^{-1-\frac{\alpha+\hat{\beta}_0}{\hat{\beta}_1\alpha}} \frac{(1 - e^{-\frac{N}{\alpha}}\eta)}{1-\eta}.$$

Δ^* is an increasing function of χ. To check monotonicity of Δ^* it suffices to check monotonicity of χ. Therefore note that the Lambert function obeys

$$\tfrac{d}{dw}\Phi_{-1}(w) = \tfrac{\Phi_{-1}(w)}{w(1+\Phi_{-1}(w))} > 0$$

for $w \in (-1/e, 0)$ (see Corless et al. (1996)). For the sake of brevity, we confine our proof to the statement one and four, the rest can be derived in a similar manner. With respect to order size, we obtain

$$\begin{aligned}\frac{d\chi}{dN} &= \alpha\frac{d\Phi_{-1}(w(N))}{dN}\\ &= \alpha\frac{\Phi_{-1}(w)}{w(1+\Phi_{-1}(w))}\frac{dw}{dN} = \alpha\frac{\Phi_{-1}(w)}{w(1+\Phi_{-1}(w))}\left(-e^{-1-\frac{\alpha+\hat{\beta}_0}{\hat{\beta}_1\alpha}-\frac{N}{\alpha}}\frac{\eta}{1-\eta}\right) < 0.\end{aligned}$$

With respect to η, we obtain

$$\begin{aligned}\frac{d\chi}{d\eta} &= \alpha\frac{d\Phi_{-1}(w(\eta))}{d\eta}\\ &= \alpha\frac{\Phi_{-1}(w)}{w(1+\Phi_{-1}(w))}\frac{dw}{d\eta} = \alpha\frac{\Phi_{-1}(w)}{w(1+\Phi_{-1}(w))}\left(-e^{-1-\frac{\alpha+\hat{\beta}_0}{\hat{\beta}_1\alpha}}\frac{1-e^{-\frac{N}{\alpha}}}{1-\eta}\right) < 0,\end{aligned}$$

□

Proof of Proposition 6. Check that for $\eta = 1$, W as of (2.2.2) reduces to

$$W = \mu\left(1 - pe^{-\frac{D}{\alpha}}\left(1 - e^{-\frac{N}{\alpha}}\right)\left(\alpha - \frac{\alpha\hat{\beta}}{\alpha+\hat{\beta}}\right)\right). \tag{2.A.8}$$

With only α and $\hat{\beta}$ depending on I, the derivative of W with respect to Δ reads

$$\begin{aligned}\frac{dW}{d\Delta} = -\frac{p\mu e^{-\frac{D+N}{\alpha}}}{(\alpha+\hat{\beta})^2}\Bigg\{&\frac{\partial\alpha}{\partial I}\left((e^{\frac{N}{\alpha}}-1)\left(D(\alpha+\hat{\beta}) + \alpha^2 + 2\hat{\beta}\alpha\right) - N(\alpha+\hat{\beta})\right)\\ &+ \frac{d\hat{\beta}}{dI}\alpha^2(e^{\frac{N}{\alpha}}-1)\Bigg\}\frac{dI}{d\Delta}.\end{aligned} \tag{2.A.9}$$

Since by construction $\frac{\partial I}{\partial\Delta} > 0$ and $\hat{\beta} \geq 0$ and by assumption - and to avoid trivial cases - $p > 0$, $\mu > 0$, $\alpha > 0$ hold, $\frac{\partial W}{\partial\Delta} = 0$ is equivalent to (2.2.32). □

Appendix 2.B Descriptive Statistics

Table 2.1: Estimates of unconditional order flow probabilities according to (2.2.5)-(2.2.8). $1-q$, $1-p$, $1-\hat{q}$ report the respective probabilities that no limit order arrives at the submission (i.e. best bid) price level, that no limit order arrives at more aggressive levels and that no market order arrives. $1-r$ refers to the probability that at t_0 there is no hidden depth at the submission price level. Estimates for the first three order flow probabilities have been undertaken for the periods of $3s, 10s$ and $30s$.

Stock	Δt (in sec.)	Order Flow Probability			Hidden Depth Probability
		$1-q$	$1-\hat{q}$	$1-p$	$1-r$
AAPL	3	0.12	0.44	0.26	0.47
	10	0.05	0.24	0.04	
	30	0.03	0.14	0.00	
AMZN	3	0.35	0.57	0.57	0.46
	10	0.15	0.33	0.24	
	30	0.07	0.18	0.03	
CSCO	3	0.09	0.97	0.67	0.43
	10	0.02	0.87	0.37	
	30	0.02	0.71	0.09	
DELL	3	0.19	0.98	0.77	0.34
	10	0.04	0.89	0.52	
	30	0.01	0.72	0.24	
EBAY	3	0.23	0.98	0.75	0.49
	10	0.05	0.90	0.46	
	30	0.02	0.73	0.11	
HPQ	3	0.18	0.79	0.64	0.68
	10	0.06	0.61	0.34	
	30	0.03	0.43	0.08	
MSFT	3	0.11	0.96	0.67	0.43
	10	0.02	0.85	0.37	
	30	0.01	0.67	0.10	
ORCL	3	0.12	0.97	0.69	0.49
	10	0.02	0.88	0.42	
	30	0.01	0.72	0.13	

Table 2.2: Sample statistics and unconditional parameter estimates for the periods $3s$, $10s$ and $30s$. The table reports unconditional expectation values for price, the spread (ticks as well as in basis-points relative to the actual quoted mid-point-price), the best bid depth or top-of-the-book (D_{bid}), the period's total buy-side trade volume ($E[x]$), the trade size ($E[x|x > 0]$), the limit order flow volume at aggressive price levels ($E[\hat{y}]$), the net-flow volume at aggressive price level ($E[x - \hat{y}]$), the limit order flow volume at prevailing best bid (ask) price levels ($E[y]$), the top-of-book cancellation ratio (c) and finally the ratio between average top of book depth and the average trade size ($\frac{D_{bid}}{\alpha}$).

	Δt (sec)	$price$	$spread$ $(ticks)$	$spread$ (bps)	D_{bid} $(TopBook)$	$\alpha \cdot p$ $(E[x])$	α $(TrSize)$	$\hat{\beta} \cdot \hat{q}$ $(E[\hat{y}])$	$\alpha p - \hat{\beta}\hat{q}$ $E[x-\hat{y}]$	$\beta \cdot q$ $(E[y])$	c $(Cancel)$	$\frac{D_{bid}}{\alpha}$
AAPL	3	88.81	1.92	2.16	505	640	863	401	239	311	0.53	4.12
	10	88.8	1.92	2.17	501	1952	2024	1174	778	509	0.61	1.23
	30	88.8	1.93	2.17	499	5572	5573	3233	2339	803	0.65	0.15
AMZN	3	53.3	2.91	5.46	324	215	494	153	62	138	0.40	4.03
	10	53.3	2.92	5.48	322	611	805	481	130	233	0.54	2.31
	30	53.3	2.93	5.50	324	1730	1784	1336	394	374	0.61	0.77
CSCO	3	16.68	1.02	6.13	19776	932	2860	242	690	3753	0.18	128.43
	10	16.68	1.03	6.16	19741	2708	4287	2391	317	7267	0.34	89.29
	30	16.68	1.03	6.16	19686	7764	8548	7700	64	12118	0.50	48.35
DELL	3	10.54	1.05	9.93	8520	600	2617	63	537	1185	0.12	79.76
	10	10.54	1.05	9.96	8498	1477	3098	651	826	2377	0.23	63.10
	30	10.54	1.05	9.98	8480	3682	4835	2219	1463	4234	0.39	51.70
EBAY	3	14.32	1.03	7.21	5904	234	925	28	206	1070	0.16	47.96
	10	14.31	1.04	7.25	5824	643	1192	440	203	2016	0.32	33.55
	30	14.31	1.04	7.28	5840	1746	1957	1713	33	3783	0.52	20.64
HPQ	3	36.33	1.27	3.49	945	181	500	150	31	422	0.45	5.14
	10	36.33	1.27	3.50	942	501	756	388	113	641	0.62	3.89
	30	36.33	1.27	3.51	942	1433	1557	951	482	904	0.72	2.00
MSFT	3	19.99	1.03	5.15	22213	956	2902	519	437	3632	0.17	122.58
	10	19.99	1.03	5.17	22216	2716	4338	3141	-425	6904	0.31	89.82
	30	19.98	1.04	5.18	22140	7470	8309	9346	-1876	11819	0.48	46.69
ORCL	3	18.09	1.03	5.69	13022	887	2834	173	714	2452	0.17	76.63
	10	18.09	1.03	5.70	12938	2479	4255	1477	1002	4909	0.32	53.13
	30	18.09	1.03	5.71	12920	6680	7659	5198	1482	8907	0.50	28.82

Table 2.3: Correlation estimates between (initial) imbalance I_0 and the conditional flow volume and price averages for the time periods $\Delta t = 3s, 10s, 30s$. We report estimates for each stock separately. Since Apple and Amazon are stocks that on average trade at wider spreads, we provide two estimates, one representing small spread scenarios (.25-quintile) and the other representing large ones (.75-quintile). For Cisco, Dell, Ebay, Hewlett-Packard, Microsoft and Oracle we only provide one estimate, since they basically trade at one-tick spreads, see also table 2.2. Results indicate how significantly the (initial) imbalance affects future expected order flow volumes as well as expected price. The 0.001, 0.01 and 0.1 level of significance are denoted by $(^{**})$,$(^{*})$ and $(^{-})$ respectively.

	$\Delta t = 3s$				$\Delta t = 10s$				$\Delta t = 30s$			
	$E[y_I]$	$\boldsymbol{E}[\hat{\boldsymbol{y}}_{\boldsymbol{I}}]$	$E[x_I]$	$\boldsymbol{\mu_I}$	$E[y_I]$	$\boldsymbol{E}[\hat{\boldsymbol{y}}_{\boldsymbol{I}}]$	$E[x_I]$	$\boldsymbol{\mu_I}$	$E[y_I]$	$\boldsymbol{E}[\hat{\boldsymbol{y}}_{\boldsymbol{I}}]$	$E[x_I]$	$\boldsymbol{\mu_I}$
AAPL(a)	-0.05^{**}	$\mathbf{0.09}^{**}$	0.02^{**}	$\mathbf{0.09}^{**}$	-0.03^{**}	$\mathbf{0.09}^{**}$	0.02^{*}	$\mathbf{0.06}^{**}$	0.00^{-}	$\mathbf{0.06}^{**}$	0.00^{-}	$\mathbf{0.04}^{**}$
AAPL(b)	0.02^{**}	$\mathbf{0.13}^{**}$	0.03^{**}	$\mathbf{0.11}^{**}$	0.04^{**}	$\mathbf{0.09}^{**}$	0.03^{**}	$\mathbf{0.06}^{**}$	0.03^{**}	$\mathbf{0.05}^{**}$	0.02^{**}	$\mathbf{0.03}^{**}$
AMZN(a)	0.03^{*}	$\mathbf{0.03}^{*}$	0.00^{-}	$\mathbf{0.11}^{**}$	0.06^{**}	$\mathbf{0.09}^{**}$	-0.03^{*}	$\mathbf{0.11}^{**}$	0.07^{**}	$\mathbf{0.08}^{**}$	-0.04^{**}	$\mathbf{0.07}^{**}$
AMZN(b)	0.06^{**}	$\mathbf{0.09}^{**}$	0.00^{-}	$\mathbf{0.10}^{**}$	0.03^{**}	$\mathbf{0.06}^{**}$	-0.02^{*}	$\mathbf{0.07}^{**}$	0.03^{**}	$\mathbf{0.06}^{**}$	-0.03^{**}	$\mathbf{0.07}^{**}$
CSCO	-0.07^{**}	$\mathbf{0.12}^{**}$	-0.06^{**}	$\mathbf{0.23}^{**}$	-0.04^{**}	$\mathbf{0.12}^{**}$	-0.05^{**}	$\mathbf{0.20}^{**}$	0.02^{**}	$\mathbf{0.07}^{**}$	-0.04^{**}	$\mathbf{0.12}^{**}$
DELL	0.01^{*}	$\mathbf{0.16}^{**}$	0.00^{-}	$\mathbf{0.15}^{**}$	0.03^{**}	$\mathbf{0.20}^{**}$	0.00^{-}	$\mathbf{0.17}^{**}$	0.03^{**}	$\mathbf{0.21}^{**}$	0.01^{-}	$\mathbf{0.14}^{**}$
EBAY	0.01^{-}	$\mathbf{0.16}^{**}$	-0.07^{**}	$\mathbf{0.25}^{**}$	0.04^{**}	$\mathbf{0.19}^{**}$	-0.07^{**}	$\mathbf{0.24}^{**}$	0.05^{**}	$\mathbf{0.15}^{**}$	-0.07^{**}	$\mathbf{0.17}^{**}$
HPQ	0.02^{**}	$\mathbf{0.12}^{**}$	-0.03^{**}	$\mathbf{0.19}^{**}$	0.04^{**}	$\mathbf{0.11}^{**}$	0.00^{-}	$\mathbf{0.13}^{**}$	0.06^{**}	$\mathbf{0.08}^{**}$	0.01^{*}	$\mathbf{0.06}^{**}$
MSFT	-0.12^{**}	$\mathbf{0.10}^{**}$	-0.07^{**}	$\mathbf{0.22}^{**}$	-0.06^{**}	$\mathbf{0.10}^{**}$	-0.04^{**}	$\mathbf{0.20}^{**}$	0.04^{**}	$\mathbf{0.10}^{**}$	0.05^{**}	$\mathbf{0.12}^{**}$
ORCL	-0.08^{**}	$\mathbf{0.13}^{**}$	-0.10^{**}	$\mathbf{0.21}^{**}$	-0.07^{**}	$\mathbf{0.12}^{**}$	-0.08^{**}	$\mathbf{0.18}^{**}$	0.02^{**}	$\mathbf{0.09}^{**}$	-0.05^{**}	$\mathbf{0.12}^{**}$

Figure 2.3: Order flow parameter estimates with respect to the order book imbalance I. Estimation is based on weighted OLS of cubic polynomials. Absolute expected order flow volume for the three flow types x, $\hat{y}$, y normalised with respect to $I = 0$.

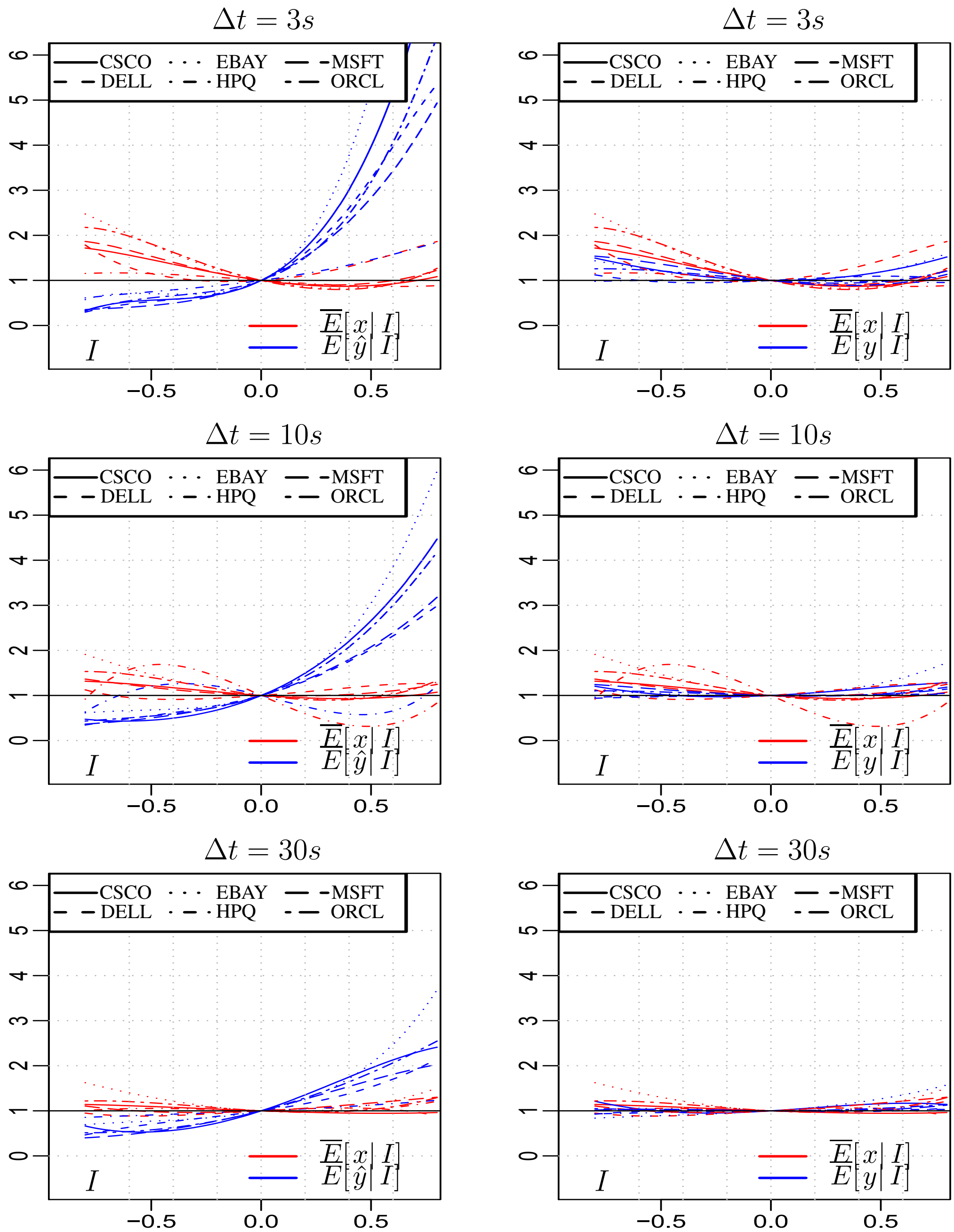


Figure 2.4: Estimation results for the conditional price impact $E\left[A_T|\, I\right]$ with respect to the order book imbalance I. Estimation is based on weighted OLS of cubic polynomials.

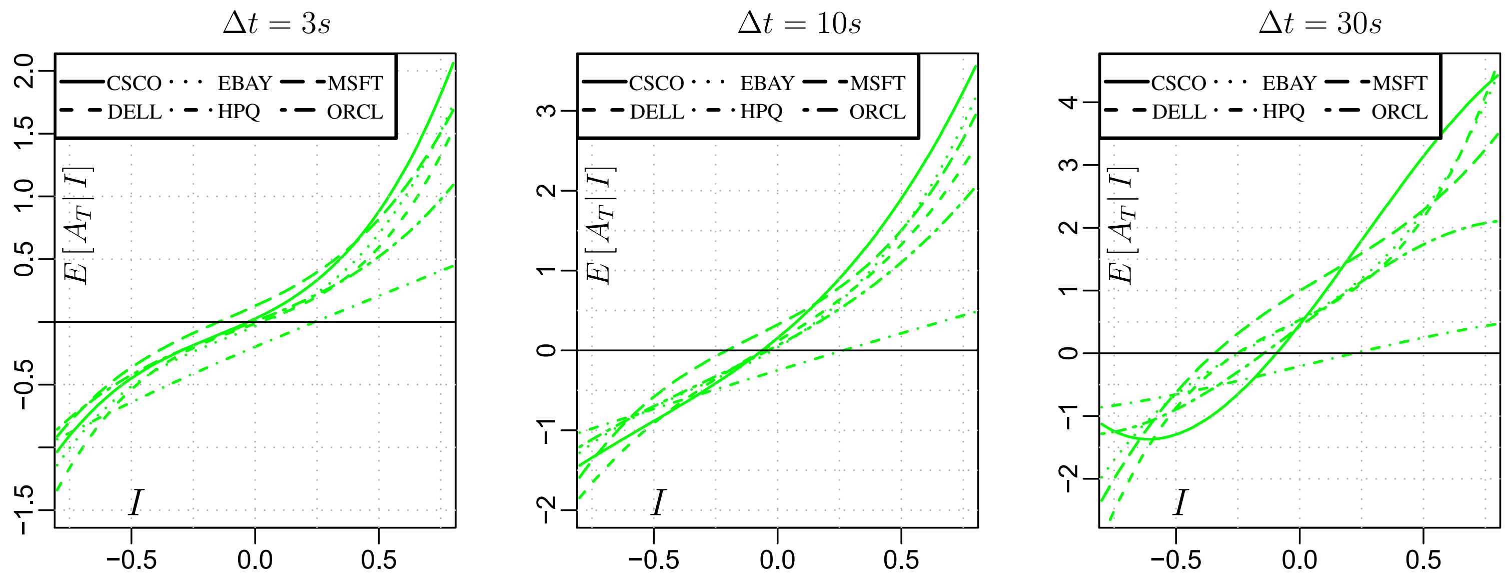


Figure 2.5: Estimates of the optimal exposure ratio (Δ_r^*) for Apple, Amazon, Cisco and Dell are shown for the case $\Delta t = 10s$ and the spread equaling one cent. (Cumulative) Order sizes are given in shares. The optimal display ratio is computed against the initial order book imbalance (I_0) and the initial best-bid depth (D_{bid}).

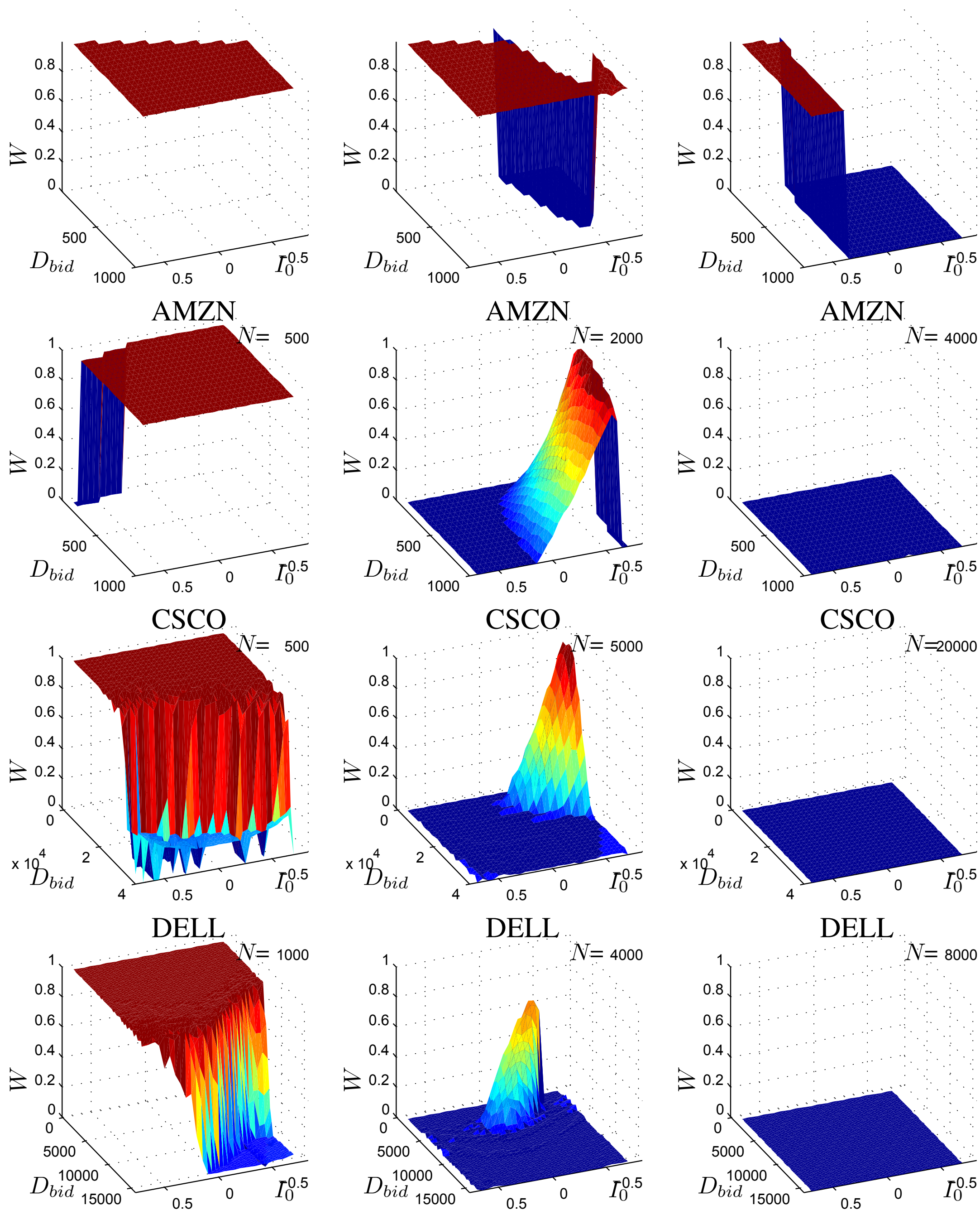


Figure 2.6: Estimates of the optimal exposure ratio (Δ_r^*) for Ebay, Hewlett-Packard, Microsoft and Oracle are shown for the case $\Delta t = 10s$ and the spread equaling one cent. (Cumulative) Order sizes are given in shares. The optimal display ratio is computed against the initial order book imbalance (I_0) and the initial best-bid depth (D_{bid}).

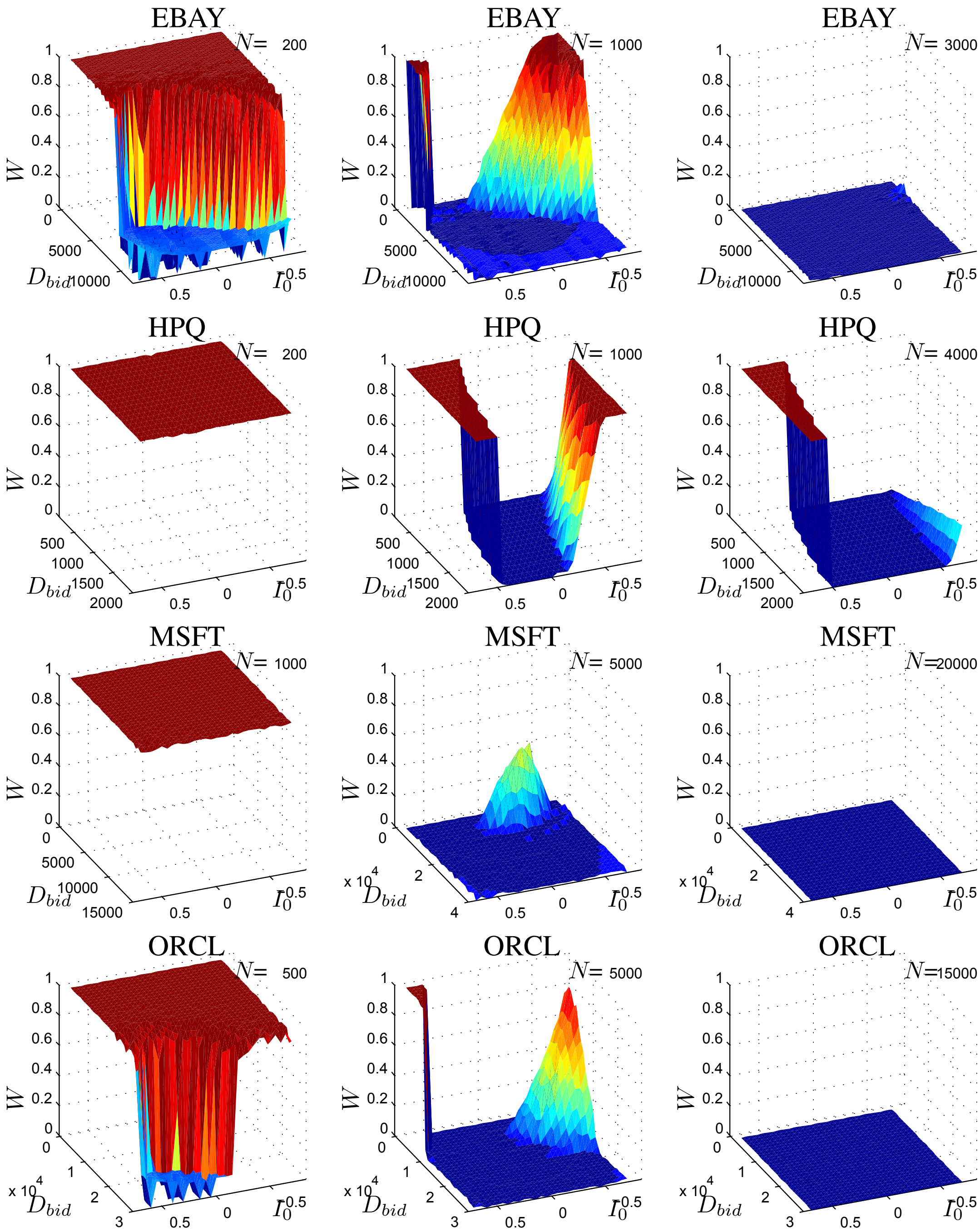


Figure 2.7: Comparison between a hidden strategy with Iceberg against a visible limit order σ. We fixed the Icebergs trading horizon ($\Delta t = 30s$), side-of-trade (buy), spread (1 cent), as well as its overall size in shares(N). For realized (initial) order book imbalance $I$and queue size D_{bid} in shares, we display the Iceberg orders potential trade improvement σ.

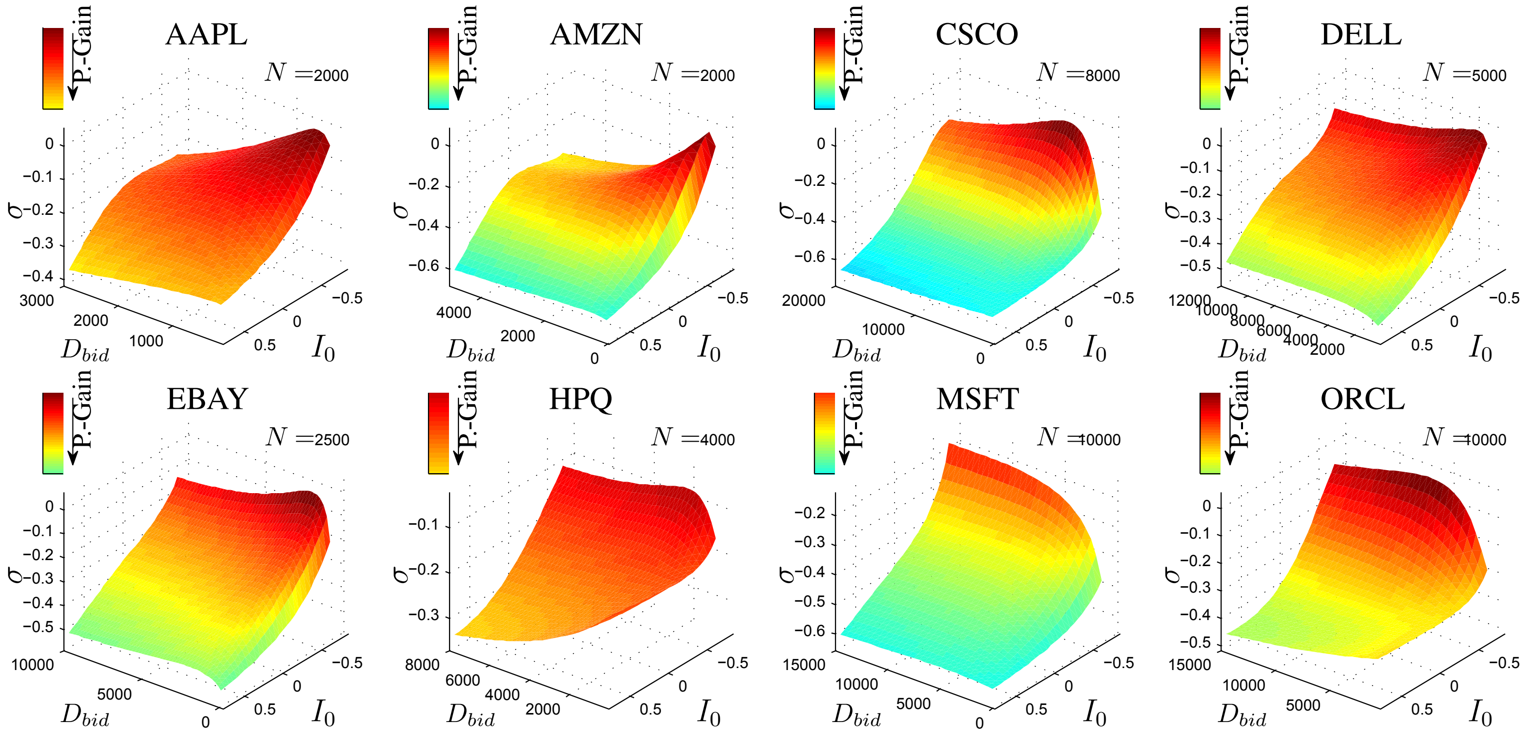
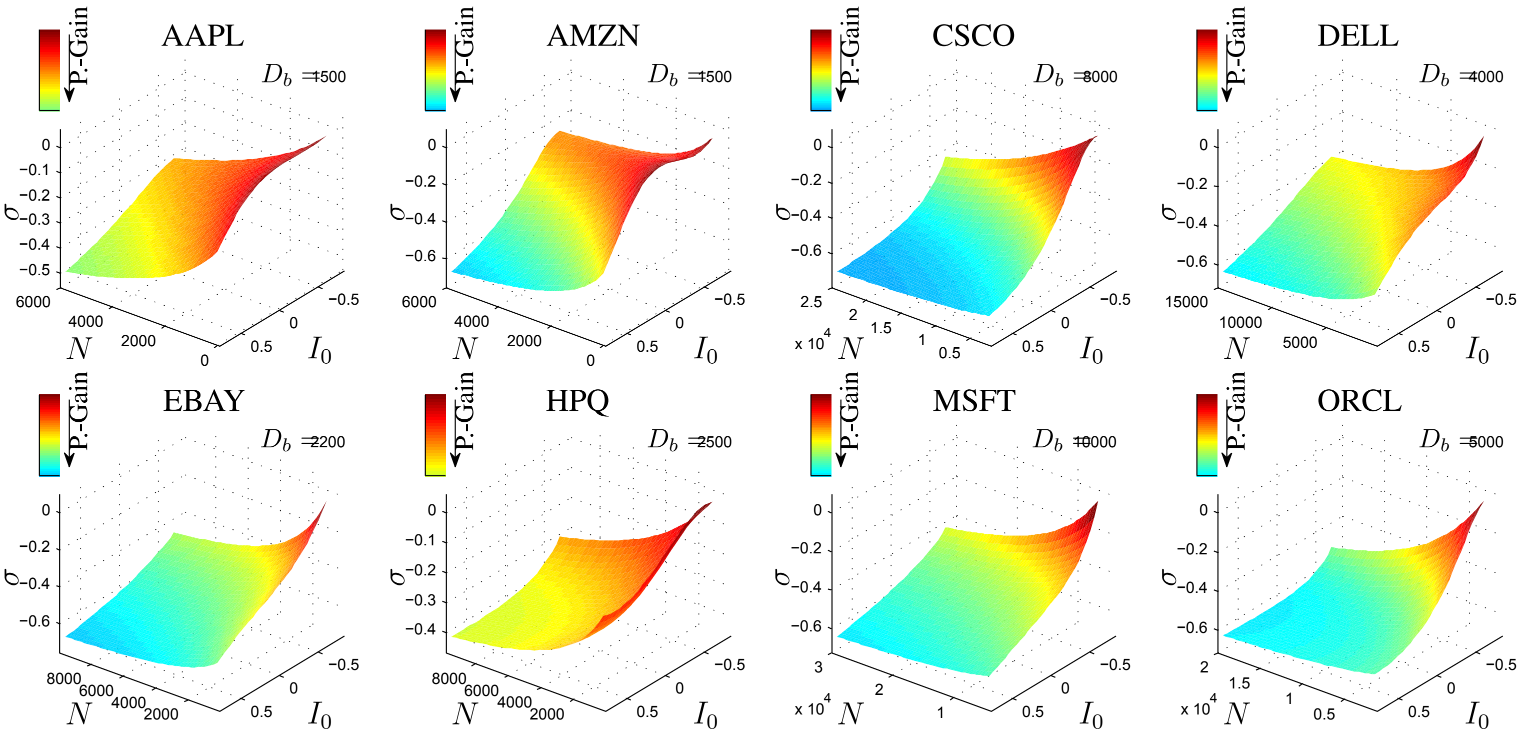


Figure 2.8: Comparison between a hidden strategy with Iceberg against a visible limit order σ. We fixed the Icebergs trading horizon ($\Delta t = 30s$), trade size (buy), spread (1 tick), as well as the best bid depth D (for shorter notation we have written D_b instead of D_{bid}). For realized (initial) order book imbalance (I_0) and Icebergs order size (N) we display the Iceberg orders potential trade improvement σ.



Chapter 3

A Trading Game of Hidden Liquidity Supply under Latent Demand

This chapter is based on Cebiroglu et al. (2012).

3.1 Introduction

Hidden liquidity has transformed into of the major issues in financial markets research. One of the reasons is that the proliferation of hidden liquidity has seen a dramatic increase over the past decade and nowadays accounts for a substantial proportion of trading activity and volume.[1] Yet, despite its growing role in trading practice, academic insights are still few and hidden liquidity remains a subject of a highly controversial debate. One central issue is which factors contribute in the origination of hidden liquidity; and why certain markets trade more *dark* than others. A second and maybe more relevant issue: regulators and exchange operators are increasingly worried about the possible downsides of dark trading. Although the increased use of hidden orders suggests that a growing number of investors perceives a tangible benefit, they argue that this might come at the cost of low market quality and worse price efficiency.

This paper contributes to the ongoing debate as follows. We propose a trading game that captures several key features of modern trading mechanisms: liquidity competition, counterparty attraction, liquidity externalities and latent trade demand. Our framework allows to address the above issues in two ways. First, we identify key market factors and motifs that lead to hidden order submissions; spread, tick size, order imbalance and block trading commission fees - they all - affect the amount of hidden liquidity. And second, we investigate the effect of hidden order submissions on the wider market along several dimensions of market quality.

[1]For instance, recent studies report that hidden volume accounts for 25%, 44%, 27% and 20.4% of total liquidity on the Euronext-Paris, in NASDAQ National Market Quotes, the french CAC40 and the Belgian BEL20 respectively, see Tuttle (2003), D'Hondt et al. (2004), De Winne and D'Hondt (2007, 2009).

For this purpose, we propose a dynamic equilibrium framework with two markets: an *upstairs* and a *downstairs* market. This feature accommodates the realities of the modern trading landscape. In fact, literature suggests that the presence of upstairs trading mechanisms alongside a public primary market makes markets even more efficient compared to single exchange markets. This is known as the *liquidity externality puzzle*. In particular, upstairs markets are more cost-efficient in transacting large *block trades*, while downstairs markets contribute to discovery and provide liquidity.[2] The downstairs market is *order-driven* and enforces a strict *price-time* precedence rule among outstanding limit orders. The upstairs market on the other hand, operates as a brokerage market where agents conduct counterparty search on behalf of their block trading clients.

In our model, order arrival is sequential and all traders are subject to liquidation constraints within a finite time horizon. In the downstairs market, the model features a liquidity demanding noise trader and two traders who compete in liquidity provision. A forth trader, a large block trader, has discretion over the trading place: he can either consume liquidity provided by the downstairs traders or *shop* the block upstairs. The block trader is a *latent* or *opportunistic* trader in the sense that he does not continuously and publicly express his trading demand, since it is costly (c.f. Grossmann (1992)). Instead, he trades upon observable liquidity opportunities provided by other market participants. For the latent block trader, trading in the upstairs market is advantageous out of two reasons. First, trades do not impact public (i.e. downstairs) prices. Second, the upstairs market provides more liquidity. The downside of trading upstairs is related to brokerage fees and compensation for counterparty search services.[3] Downstairs markets on the other hand, are more cost-efficient when they provide enough liquidity, i.e. *critical mass*. Ultimately, the block trader has to find the trade-off between trading in the upstairs and trading in the downstairs market.

Downstairs' liquidity providers are heterogeneous in their trading style: the *hidden trader* has discretion over the display/hidden size, while the *liquidity competitor* has discretion over the trading price. To reduce execution risk, the liquidity competitor has an incentive to undercut a (visible) large order, i.e. submit a more aggressive order. The hidden trader on the other hand, can prevent this from happening by using hidden orders. However, as the latent block trader only trades in the public market when he observes a *critical size* of (observable) liquidity, a reduction of visible liquidity will also less likely attract the latent block trader to act as a counterparty.

Under several simplifying assumptions, we derive analytic expressions for the (i) equilibrium trading strategies and (ii) equilibrium market properties, including market volatility and the level of hidden liquidity supply. The hidden trader's order display decision is affected by his order size, the presence of the latent block trader and the market conditions. If the hidden trader does not have the *critical size* or the presence of the latent trader is unlikely, hidden traders choose the display size that is just enough to avoid liquidity competition. The degree of liquid-

[2]In fact, various theoretic studies suggest that *downstairs markets need an upstairs market*, see for instance Grossmann (1992), Seppi (1990), Madhavan and Cheng (1997) and Keim and Madhavan (1996). For empirical evidence, see Griffiths et al. (2001, 1998), Bessembinder and Venkatamaran (2004), Booth et al. (2002).

[3]In reality, the delay risk of finding a counterparty does also play a role. We do not account for this aspect in our model.

ity competition or *order aggressiveness* depends on several market factors. When the spread is wide, the (relative) tick size is small, the depth on the opposite side of the book is low and the depth on the same side is high, same-side liquidity competition increases. Accordingly, markets observe a higher degree of hidden liquidity supply when markets have wider spreads and smaller relative tick sizes and hidden liquidity concentrates always on the heavy side of the book.

On the other hand, in the presence of latent block trader demand and when the hidden trader has the *critical size* to attract the block trader, the hidden trader will expose his full trading intentions. In this case, exposure mutually benefits all market participants, as counterparties search is facilitated. If the (large) hidden trader would instead use hidden orders, the latent trader and the hidden trader would miss out on mutually beneficial trades.

The latter scenario has important implications with respect to the efficiency of prices. We argue that large hidden orders cause frictions by exacerbating excess returns and market volatility. The reason is as follows. If large downstairs traders hide their trading intentions, they will less likely attract latent counterparties of the right size. The consequence is that these traders are more likely to cancel their orders and enforce liquidation by using costly market orders. Since large market orders cause price impact, price pressures and price fluctuations arising from standing hidden orders must be disproportionally larger than for standing displayed orders. In essence, the result reflects the view that price fluctuations arise due to *liquidity effects* of asynchronous order arrivals. When trader's display their trading intentions these temporal imbalances can be quickly absorbed by attracting latent counterparties. This is closely related to the benefits of *sunshine trading* according to Admanti and Pfleiderer (1991) and *synchronization* issues in the *theory of bubbles* as in Abreu and Brunnermeier (2003).

One important consequence is that based on these liquidity or coordination effects, we establish a theoretic linkage between several key observable market characteristics and hidden liquidity. In particular, our predictions are in line with Cebiroglu and Horst (2011) (see chapter 1). Our theory predicts that wider spreads cause an increased hidden liquidity supply. And due to coordination frictions, hidden liquidity increases market volatility. In particular, we conjecture that the causal link between spread and volatility partly arises due to frictions and that hidden liquidity is the middle-man of this causation.

Our theory differs from the information-based literature (c.f. Glosten and Milgrom (1985) and Copeland and Galai (1983)). First, they link spread and volatility based on the concept of informational asymmetry. Second, they predict the opposite causation, i.e. larger volatility increases wider spreads such that market makers compensate for the risk of getting *picked-off*.

To empirically test the predictions of our model, we perform an extensive empirical analysis utilising two unique datasets. First, the presence of hidden orders is inferred from the NASDAQ Model-View dataset, providing one-minute snapshots of the entire (inclusive hidden) limit order book of NASDAQ stocks. Second, order flow data (cancellations, submissions and executions) is obtained by reconstructing high-frequency message level data from NASDAQ provided by the data interface *Lobster*. Our empirical analysis tests the model predictions along the cross-sectional and dynamic dimension. We use an extensive set of 448 stocks from the S&P 500 that were continuously traded in the month of January 2009, to test whether 1) hidden liquidity provision is affected by the tick size and the size of the spread and 2) how hidden liquidity provision affects volatility. The estimation results are in line with our theory. In particular, hidden liquidity provision increases with the size of the spread and the inverse of the tick size. Moreover, volatility is positively affected by hidden liquidity provision (and indirectly by tick and spread size).

Second, we model high-frequency market dynamics using an vector auto-regressive model and test the theoretical predictions based on (generalised) impulse response functions arising from shocks in bid-ask (hidden and displayed) depth imbalances. We report statistically significant market reactions after periods of high (one-sided) hidden depth is reported. Controlling for other order book variables and multivariate market dynamics we find hidden depth causing significant excess returns and market order activity on the opposite side of the market. In line with our theoretical predictions, these effects rise with the extent of (one-sided) hidden order supply but is statistically not identifiable for displayed volume.

Our work distinguishes from earlier theoretical models as it establishes a link between three separate strands of research: the literature on *limit order book models*, *upstairs block trading* and *hidden liquidity models*. At the heart of most theoretical models lies information asymmetry. In contrast, our work is more in line with the framework proposed in Foucault et al. (2005), i.e. we focus on mechanisms and trading frictions that do not arise from informational asymmetry alone. Our work is closely related to the models of hidden order submission in Buti and Rindi (2013) and Moinas (2010). However, they do not account for liquidity externalities and competition between public exchanges and off-exchange trading mechanisms.

Our results are important for market regulators and exchange operators likewise. Public markets compete for order flow in an increasingly fragmented market. If they lose this "battle for liquidity", the public price formation process can be harmed. The literature suggests that these liquidity externalities are closely related to market transparency (e.g., Hendershott and Jones (2005) and Hendershott and Mendelson (2002)). In this work, we show that transparency can benefit public market quality if downstairs markets are *thick*. Hence, providing incentives for liquidity provision can be helpful in increasing the share and quality of public exchanges.

The remainder of this paper is structured as follows. The model is introduced in section 3.2. Equilibrium results follow in section 3.3. All the main theoretical implications and testable predictions are derived in this section. We first analyse a baseline version without large block traders and derive conclusions on various aspects of liquidity competition. At a second stage, we extent our analysis to include upstairs markets. Our main implications are tested and empirically verified in section 3.4. Section 3.5 concludes.

3.2 The Model

We introduce a sequential trading model with discrete timing. The institutional framework consists of two markets: a downstairs and an upstairs market. The trading population of the downstairs market consists of a variety of liquidity supplying and demanding participants: a hidden trader, a liquidity competitor and a noise trader. Large block investors can alternatively trade in the liquid upstairs market. The block trader is *latent* (or equally opportunistic), i.e., he continuously monitors the market and only trades upon liquidity opportunities.

3.2.1 Institutional Framework

The downstairs market is *order-driven*, i.e., investors can openly quote prices and trade against the public order book. However, because some orders may be hidden, not all liquidity is fully disclosed to the open public. The upstairs market, on the other hand, accomplishes trading through a private network of broker dealers and trading desks. Brokers act as intermediaries and locate suitable counterparts for their clients. While trading in downstairs markets is generally characterised by smaller-sized trades, upstairs markets appeal to large institutional investors as they facilitate large block transactions (see, e.g., Madhavan and Cheng (1997), Keim and Madhavan (1996)).

The Downstairs Market

Orders are submitted on a discrete price grid with a minimum price variation (i.e., tick size) Δ. Prices at time t are given relative to the prevailing best ask A_t and bid B_t price. To keep the model tractable, we assume a *competitive* or *liquid* market, where the *spread resiliency*, i.e., the speed of reversion of spreads to their equilibrium level (see, e.g., Foucault et al. (2005)), is higher than the time scale underlying our model. This is in line with Kyle (1985) and Glosten and Milgrom (1985) and a realistic assumption for liquid markets.[4]

Assumption 1. *Market makers are competitive such that the spread instantly reverts back to the competitive level S, once a change occurred, i.e.,*

$$S_t = S \qquad \forall\, t > 0. \tag{3.2.1}$$

In particular, this assumption implies that order submissions, cancellations and executions on one side of the market are instantly corrected by price revisions on the opposite side, such that the spread always remains at its equilibrium value. We assume that limit orders can be submitted either on the best bid (ask) quote or with a price improvement of one tick (i.e., in the spread). Limit orders are executed against incoming market orders in a discriminatory way using a hierarchy of (i) price priority, (ii) display priority and (iii) time priority. Suppose a buy (hidden) limit order trader has a trading horizon until time τ and enters the market at time t aiming at buying N shares at the (submission) price p_t^l. Moreover, in line with Harris and Hasbrouck (1996), we assume that limit orders pose a *precommitment* to trade. Therefore, at terminal time τ, non-executed shares need to get cancelled and turned into market orders to guarantee trade execution. Consequently, after normalising by the "arrival price" B_tN, the trader's "implementation shortfall" according to Perold (1988), Bessembinder et al. (2009), Harris and Hasbrouck (1996) is given by

$$\Pi_\tau := \underbrace{\left(p_t^l - B_t\right) V_\tau}_{\text{Executed limit order}} + \underbrace{\left(p_\tau^m - B_t\right)\left(N - V_\tau\right)}_{\text{Unexecuted Limit Order}}, \tag{3.2.2}$$

[4]It is known that the speed of resiliency is linked to the liquidity of a market, see Biais et al. (1995), Degryse et al. (2005), Domowitz and Madhavan (2003)

where p^m_τ denotes the price of the market order and V_τ the executed limit order until time τ. While the trader's limit order submission price p^l_t is a matter of choice, the market order price p^m_τ is determined by the order's price impact and thus depends on the execution volume and the prevailing depth. In line with Obizhaeva and Wang (2013) and to keep the analysis tractable, we assume the price impact being linear in the remaining shares, i.e.,

$$p^m_{t+\tau} := B_{t+\tau} + \underbrace{S + \frac{1}{2}\beta(N - V_{t+\tau})}_{\text{Effective Spread}}. \tag{3.2.3}$$

Finally, we define "market volatility" as the conditional variance of the cumulative price change measured from t to τ,

$$Var_t[R_\tau] := \mathbb{E}_t[(R_\tau - \mathbb{E}_t[R_\tau])^2], \tag{3.2.4}$$

with price changes R_τ given by $R_\tau := P_\tau - P_{t_0}$ and P_t denoting the mid-quote at t, i.e. $P_t := \frac{A_t + B_t}{2}$. Returns are obtained by normalising R_t by P_{t_0} with the latter being fixed at t_0.

The Upstairs Block Trading Market

In upstairs markets, investors employ the services of brokers negotiating with clients a settlement price depending on the liquidity needs and the state of the market. Consistent with Keim and Madhavan (1996), Booth et al. (2002), Harris (2003), we assume that price formation in the upstairs markets consists of two components: a (reference-) price from the public downstairs market and a commission fee γ compensating the broker's counterparty search costs. Then, the upstairs settlement price p^γ_t can be expressed as

$$p^\gamma_t = \begin{cases} A_t + \gamma & \text{if buyer,} \\ B_t - \gamma & \text{if seller.} \end{cases} \tag{3.2.5}$$

We implicitly assume that the upstairs market provides an infinite liquidity reservoir, providing immediate and guaranteed execution at price p^γ_t.

3.2.2 Market Participants and Timing

We consider four market participants arriving in sequential order: a *hidden trader* arriving at t_0, a *liquidity competitor* (t_1), a *latent* or *upstairs* block trader (t_2) and a *noise trader* (t_3). We denote the respective strategies of each trader by σ_H, σ_C and σ_L. The timing of events is depicted in Figure 3.1. The noise trader represents exogenous and random liquidity demand, i.e., a market order with size x. For simplicity, we assume the order size x being exponentially distributed with mean λ. The remaining three traders interact strategically and are risk-neutral. All participants trade out of non-informational liquidity reasons.

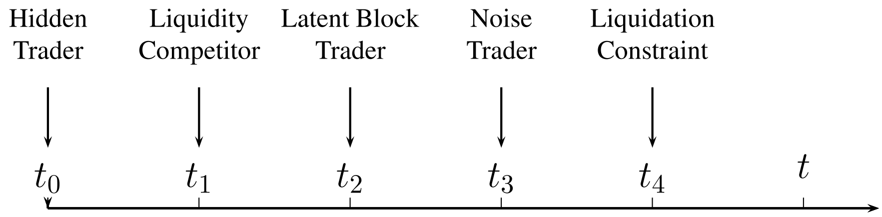


Figure 3.1: Order arrival dynamics. At terminal time t_4, all open positions hold by the downstairs limit order traders are executed via market orders.

The Downstairs Traders

The (buy) hidden trader H has N_H shares to trade until time τ. He can hide a portion of his order using an iceberg order with only D_H shares being shown to the open public, while remaining shares are kept hidden. Hence, his strategy σ_H is to decide on the magnitude of D_H. In line with the trading mechanisms in the downstairs market (see subsection 3.2.1), at arrival time t_0, he submits his order at the best prevailing bid price B_{t_0}. As the trading horizon τ is fixed, we henceforth omit the time subscript for the execution volume and replace it with the hidden trader's or liquidity competitor's specifier H and C, respectively. Accordingly, the downstairs trader's payoff according to (3.2.2) and (3.2.3) is given by

$$\Pi_H(\sigma_H, \sigma_C, \sigma_L) = \left(S + \frac{1}{2}\beta(N_H - V_H(\sigma_H, \sigma_C, \sigma_L))\right)\left(N_H - V_H(\sigma_H, \sigma_C, \sigma_L)\right). \quad (3.2.6)$$

The hidden trader's execution volume V_H and thus his payoff depends on all traders' strategies σ_C, σ_H and σ_L.

Subsequently, the liquidity competitor C arrives at t_1 with size N_C. His strategy σ_C consists of deciding on the optimal submission price level: submission at the prevailing bid price B_{t_0} ("$stay$") or at $B_{t_0} + \Delta$ ("$step$"). Accordingly, his payoff derived from (3.2.2) is

$$\begin{aligned}\Pi_C(\sigma_H, \sigma_C, \sigma_L) &= \Delta \cdot 1_{\{\sigma_C = step\}} V_C(\sigma_H, \sigma_C, \sigma_L) \\ &+ \left(\Delta \cdot 1_{\{\sigma_C = step\}} + S + \frac{1}{2}\beta(N_C - V_C(\sigma_H, \sigma_C, \sigma_L))\right)\left(N_C - V_C(\sigma_H, \sigma_C, \sigma_L)\right).\end{aligned} \quad (3.2.7)$$

The execution volumes V_H and V_C (and thereby the payoffs) depend on the trading strategies of all traders, σ_C, σ_H, σ_L, and are derived in Lemma 8 and Lemma 9 (see Appendix).

The Upstairs Block Trader

At t_2, a (sell) block trader arrives with total trade demand of N_L shares. We assume that he monitors the downstairs market with some probability δ. He has the strategic choice σ_L between trading in the downstairs market (i.e., $\sigma_L = down$) or in the upstairs market (i.e., $\sigma_L = up$). If he does not monitor the downstairs market, he only trades in the upstairs market, i.e., $\sigma_L = up$.

As the upstairs markets provides an infinite liquidity reservoir at price p_t^γ (3.2.5), there is no execution risk in this market. In contrast, liquidity supply in downstairs markets is limited and the total trade demand of block traders may not be executed. Therefore, we assume that block traders still may use the upstairs market as a last resort to enforce execution, i.e., unexecuted shares will be executed in the upstairs market.[5]

Moreover, we assume that block traders are large in the sense that their trade demand exceeds the average liquidity supply at the best price by an order of magnitude:

Assumption 2 (Block trader demand)**.** *The block trader is a large investor, i.e.,*

$$N_L > N_H + N_C. \tag{3.2.8}$$

As for large investors, continuous expression of trade demand (in terms of limit orders) in downstairs markets is costly (Grossmann (1992)), we assume that the latent block trader uses market orders only. Thus, using (3.2.5), the payoff relative to the arrival price B_{t_0} reads

$$\Pi_L(\sigma_H, \sigma_C, \sigma_L) = \begin{cases} (\Delta \cdot 1_{\{\sigma_C = step\}} - \gamma) N_L & \text{if} \quad \sigma_L = up, \\ \Delta N_C \cdot 1_{\{\sigma_C = step\}} - (\Delta + \gamma)(N_L - D_H - N_C) & \text{if} \quad \sigma_L = down. \end{cases} \tag{3.2.9}$$

We assume that the block trader does only trade the amount of shares he can really observe, i.e., that is openly displayed. Hence, he can at most trade $D_H + N_C$ shares downstairs.

The Trading Game

To provide a formal basis, we denote the action spaces of the hidden trader, liquidity competitor and the latent trader by Σ_H, Σ_C and Σ_L respectively:

$$\Sigma_H = [0, N_H], \qquad \Sigma_C = \{stay, step\}, \qquad \Sigma_L = \{down, up\}. \tag{3.2.10}$$

As both H and C are buyers, a better strategy *reduces* the payoff. Likewise, as the block trader is a seller, a better strategy *increases* his payoff.

Definition 2 (Equilibrium trading strategies)**.** *We define the triple $\sigma^* = (\sigma_C^*, \sigma_H^*, \sigma_L^*)$ constituting a (Nash-) equilibrium if for any other set of strategies $(\sigma_H, \sigma_C, \sigma_L)$ the following relations hold:*

$$\mathbb{E}_{t_0}\left[\Pi^H(\sigma_H^*, \sigma_C^*, \sigma_L^*)|\right] \leq \mathbb{E}_{t_0}\left[\Pi^H(\sigma_H, \sigma_C^*, \sigma_L^*)|\right] \tag{3.2.11}$$

$$\mathbb{E}_{t_1}\left[\Pi^C(\sigma_H^*, \sigma_C^*, \sigma_L^*)|\right] \leq \mathbb{E}_{t_1}\left[\Pi^C(\sigma_H^*, \sigma_C, \sigma_L^*)|\right] \tag{3.2.12}$$

$$\mathbb{E}_{t_2}\left[\Pi^L(\sigma_H^*, \sigma_C^*, \sigma_L^*)|\right] \geq \mathbb{E}_{t_2}\left[\Pi^L(\sigma_H^*, \sigma_C^*, \sigma_L)|\right] \quad . \tag{3.2.13}$$

[5]The fact that upstairs markets are used as a market of last resort is a common feature in institutional trading patterns. Conrad et al. (2003) show empirical evidence that 60% of block trades use upstairs market as a market of *last resort*.

The *existence* of such equilibrium in these kind of finite, dynamic games with complete information is guaranteed by *Zermelo's Theorem* and the equilibrium strategies of each player can be derived by applying the principle of *sequential rationality* or *backward-induction* (see Mas-Colell et al. (1995)). The uniqueness of equilibrium will be shown below.

3.3 Equilibrium Analysis

Solving for equilibrium in the general case $\delta \in [0, 1]$ is possible but rather tedious. Therefore, we confine our analysis to two benchmark cases providing a rich set of features and key insights. In particular, we focus on a baseline model without latent demand ($\delta = 0$), and in a second step, we include a latent block trader.

3.3.1 Equilibrium without Latent Block Traders

Under the absence of upstairs block traders, the model effectively reduces to a game of pure liquidity competition between trader H and C. We first derive the competitor's best response strategy at t_1. Subsequently, we solve for the hidden trader's equilibrium strategy.

Lemma 5 (Liquidity Competitor's Best Response)**.** *Given the hidden trader's display size D_H, the competitor's best response σ_C^* obeys*

$$\sigma_C^* = \begin{cases} stay & if \quad D_H \leq \Phi_C \\ step & else \end{cases}, \qquad \Phi_C := \begin{cases} \lambda \log\left(\frac{1}{1-g}\right) & if \quad g < 1 \\ \infty & else \end{cases} \tag{3.3.1}$$

with

$$g := \frac{N_C}{\lambda} \frac{\Delta}{S\left(1 - e^{-\frac{N_C}{\lambda}}\right) + \beta\left(N_C - \lambda\left(1 - e^{-\frac{N_C}{\lambda}}\right)\right)} \quad . \tag{3.3.2}$$

Accordingly, the liquidity competitor's aggressiveness is governed by the display or *liquidity threshold* Φ_C. An illustration of this mechanism is shown in Figure 3.2. For instance, when the (potential) hidden trader's exposure is large (i.e., $D_H > \Phi_C$), liquidity competitors are likely to "step ahead" or undercut the hidden trader's order. The reason is intuitive: for larger display sizes D_H, the competitor faces an increasing loss in time priority. To counter-balance this effect, he rises execution priority by price improvement. Moreover, liquidity competition increases for smaller tick sizes Δ, as costs for undercutting an order reduce. Finally, liquidity competition increases for wider spreads S. As wider spreads increase the costs of non-execution (i.e., opportunity costs), price improvements become more cost-efficient.[6]

Corollary 3 (Determinants of liquidity competition)**.** *Liquidity competition increases with*

[6] This follows from the fact that wider spreads cause higher market order costs when the hidden order is not fully executed at the submission price level.

(i) larger same-side depth D_H,
(ii) wider bid-ask spread S,
(iii) "thinner" opposite-side depth,
(iv) smaller tick size Δ,
(v) larger competitor demand N_C.

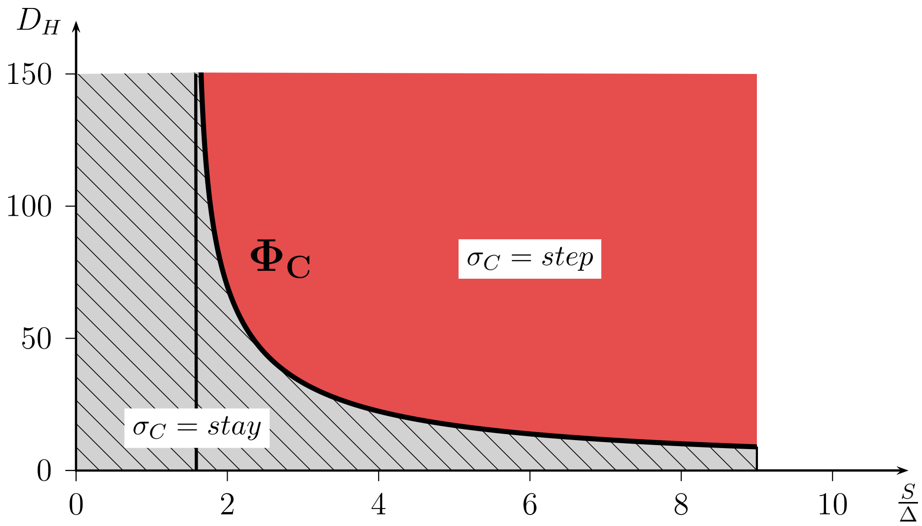


Figure 3.2: Illustration of the liquidity competition region as defined by the threshold Φ_C. The graph illustrates an example for $\lambda = N_C = 100$ shares and $\beta = 0$ (i.e., "thick" opposite side depth). When the hidden trader's display size is large, i.e., $D_H > \Phi_C$, the liquidity competitor will improve prices, $\sigma_C = step$. When the spread (tick) is wide (small), his incentive to "step ahead" increases. Φ_C diverges at $S(1 - e^{-1}) = \Delta$, i.e., liquidity competition does only payoff for stocks with $S \geq 2\Delta$.

The empirical literature on the *effects of order exposure* and *order aggressiveness* provides rich and extensive evidence for the predictions of corollary 3. For instance, in line with (i) and (ii), Cebiroglu and Horst (2013), Ranaldo (2004) and Cao et al. (2009) report that orders are more aggressive when same-side depth is large, while Biais et al. (1995), Ranaldo (2004), Cao et al. (2009), Hall and Hautsch (2006) show that liquidity competition is more likely, when the spread is wide. Likewise, empirical evidence is reported for conclusion (iii), (iv) and (v), see, for instance, Cao et al. (2009), Cebiroglu and Horst (2013) and Harris (1994, 1996, 2003), respectively.

Proposition 7 (Equilibrium)**.** *The hidden trader's and liquidity competitor's equilibrium strategies σ_H^* ($\equiv D_H^*$) and σ_C^* obey*

$$\sigma_C^* = stay, \qquad D_H^* = \begin{cases} \Phi_C & \textit{if} \quad N_H > \Phi_C \\ N_H & \textit{else} \end{cases} . \tag{3.3.3}$$

The essence of equilibrium derives mainly from Lemma 5: when the hidden trader displays more than Φ_C, the competitor undercuts the hidden trader's order to obtain (price) priority against the hidden trader. The hidden trader, however, counteracts the risk of losing price priority by limiting the display size to be below the level Φ_C that triggers the competitor's price aggressiveness. Note, however, that this exposure risk only affects large (hidden) investors, i.e., whenever $N_H > \Phi_C$. In contrast, small traders are less affected by liquidity competition.

Corollary 4 (Large orders are hidden)**.** *In equilibrium, for sufficiently large order size N_H, i.e., $N_H > \Phi_C$, the hidden trader hides his order (at least partially), i.e.,*

$$\sigma_H^* < N_H. \tag{3.3.4}$$

The fact that particularly large investors use hidden orders to reduce their exposure risks is well documented in the empirical literature. For instance, Frey and Sandas (2009) report that iceberg or hidden orders are on average 12-20 times larger than ordinary limit orders. Bessembinder et al. (2009) show that 75% of "large" orders with a notional value exceeding 50,000 EUR are at least partially hidden. They also find that 87% of the volume of large orders is hidden. From the hidden trader's equilibrium strategy D_H^* (3.3.3), the identification of the hidden liquidity determinants is straightforward.

Corollary 5 (Determinants of hidden liquidity without latent block traders)**.** *The provision of hidden liquidity increases with*

(i) larger bid-ask spreads S,
(ii) "thinner" opposite-side depth,
(iii) larger liquidity competition N_C.
(iv) smaller tick sizes Δ,

Indeed, Bessembinder et al. (2009) report that the decision to hide and the magnitude of the hidden size are positively affected by the size of the spread (confirming (i)) and negatively with opposite-side depth (confirming (ii)). Similarly, Harris (1994, 1996) and De Winne and D'Hondt (2009) report that hidden liquidity provision is low in stocks with large tick sizes (confirming (iii)). Finally, Harris (1994, 1996) suggests that the presence of liquidity competition forces traders to hide their orders (confirming (iv)).

Proposition 8 (Volatility)**.** *In equilibrium, market volatility is given by*

$$Var[R_\tau] = \beta^2\lambda^2 + \beta^2(1-q)qN_H^2. \tag{3.3.5}$$

The stochastic arrival of the noise and hidden trader induces two separate and independent sources of randomness. The randomness emanates from the arrival probability q and the noise trader's liquidity demand x with mean λ. The variance is (partly) scaled by the price impact factor β reflecting the intuition that *thinner* books give rise to higher price impact of market orders. This in turn is a source of higher variability of the stock return itself. Moreover, variance components are also scaled by the (expected) trade demand of the hidden trader N_H and the noise trader λ but not with the liquidity competitor's trade demand N_C which is deterministic. Likewise, the variance contribution of the hidden trader vanishes if his presence or absence is deterministic, i.e., $q = 0$ or $q = 1$.

3.3.2 Equilibrium with Latent Block Traders

We extend the benchmark case to an additional strategic trader, a latent block trader. This agent, is actively monitoring the downstairs market for liquidity opportunities and has discretion over the trading place: the downstairs market or the upstairs market. Because he strategically chooses between both market places, the block trader effectively introduces a mechanism of *liquidity externalities* to our model: when the downstairs market provides *cheap* liquidity opportunities, he trades downstairs. Otherwise, he sticks to the traditional market mechanism for large institutional traders, i.e. the upstairs market. Now, the hidden trader's exposure decision does not only depend on liquidity competition but also on beneficial liquidity externalities. In particular, the latent block trader poses an incentive for exposure. In the sequel of this section, we establish the equilibrium by computing first the latent investor's optimal response, then the competitor's and finally the hidden trader's recursively.

Lemma 6 (Latent trader's best response)**.** *Given, $\sigma_H = D_H$ and σ_C, the latent trader's optimal strategy σ_L^* obeys*

$$\sigma_L^* = \begin{cases} up & \textit{if} \quad 0 \le D_H < \Phi_{L_a}, \\ down & \textit{if} \quad \Phi_{L_a} \le D_H < \Phi_{L_b} \quad \textit{and} \quad \sigma_C = stay, \\ up & \textit{if} \quad \Phi_{L_a} \le D_H < \Phi_{L_b} \quad \textit{and} \quad \sigma_C = step, \\ down & \textit{if} \quad \Phi_{L_b} \le D_H, \end{cases} \tag{3.3.6}$$

with $\Phi_{L_a} \le \Phi_{L_b}$ and

$$\Phi_{L_a} := \frac{(N_L - N_C)\Delta - N_C\gamma}{\gamma + \Delta}, \qquad \Phi_{L_b} := \frac{2(N_L - N_C)\Delta - N_C\gamma}{\gamma + \Delta}. \tag{3.3.7}$$

The essence of Lemma 6 is *liquidity begets liquidity*. That is, the latent trader only trades downstairs when the hidden trader displays a sufficient amount of trade interest: either $D_H > \Phi_{L_a}$ or $D_H > \Phi_{L_b}$. The intuition is straightforward. Trading in the downstairs market is associated with price impact, whereas upstairs trading - due to the infinite liquidity reservoir - is not. Therefore, the latent trader aims at preventing adverse price impact in the downstairs market as the latter also makes trading in the upstairs market more expensive. Consequently, downstairs trading is only profitable, when large volumes can be traded such that the remaining shares (which have to be traded upstairs) are not adversely affected by the induced price impact of the prior downstairs trade.

A second and striking effect arises whenever $\Phi_{L_a} \le D_H < \Phi_{L_b}$: In this case, the liquidity competitor does not increase the chances to elicit block trade executions when providing better prices. In fact, even when $\sigma_C = step$, the block trader will trade upstairs as he only profits from trading downstairs if the liquidity supply is sufficient. Otherwise, as price improvements directly translate to the upstairs market via (3.2.5), he realises trades upstairs without price impact.

Corollary 6 (Downstairs vs. Upstairs trading)**.** *The proportion of upstairs trading increases for*

- *larger ticks Δ,*
- *lower displayed downstairs depth $D_H + N_C$,*
- *smaller upstairs commission fee γ.*

Proof. Follows immediately from the fact that block traders are more likely to refrain from trading when the displayed order size D_H exceeds the barrier Φ_{L_a} and/or Φ_{L_b}. That is more likely the case, when the commission fee γ is small, tick size Δ and total displayed depth $D_H + N_C$ is large. $\square$

For the block trader, the costs of downstairs trading is affected by the tick size, the spread and the total displayed depth. In particular, downstairs trading increases when -ceterus paribus-ticks are small, spreads are narrow, depth is large and upstairs commission fee is high. These predictions resonate well with the empirical block trading and upstairs literature. For instance, Griffiths et al. (2001) report that the proportion of upstairs trading increases when downstairs depth is low and spread is wide. The same authors report that tick size reduction increased the proportion of downstairs trading (see Griffiths et al. (1998)).

Lemma 7 (Liquidity competitor's best response)**.** *Given σ_L^* and D_H, then the liquidity competitor's best response obeys*

$$\sigma_C^* = \begin{cases} \sigma_C^{*0}(D_H) & \text{if} \quad 0 \leq D_H < \Phi_{L_a} \\ stay & \text{if} \quad \Phi_{L_a} \leq D_H \end{cases}, \tag{3.3.8}$$

with σ_C^{*0} denoting the competitor's optimal strategy in the case without the latent investor as of Proposition 5.

Because execution risk is low when the block trader trades downstairs, the competitor will never pay the extra tick Δ to improve execution priority and undercut the hidden order. In particular, whenever the hidden trader displays just enough to attract the block trader, i.e., $\Phi_{L_a} \leq D_H$, it is optimal for the competitor to *save* a tick and *stay behind* the hidden trader, i.e., (i.e. $\sigma_C = stay$). If, however, liquidity supply is low, i.e., $D_H < \Phi_{L_a}$, the block trader is not attracted to the downstairs market. In this case, the competitor's problem reduces to the baseline game without the block trader as of Proposition 7.

Proposition 9 (Equilibrium strategy)**.** *With the block trade, the equilibrium strategies σ_H^* ($\equiv D_H^*$), σ_C^* and σ_L^* obey*

$$D_H^* = \begin{cases} D_H^{*0} & \text{if} \quad N_H \leq \Phi_{L_a} \\ N_H & \text{else} \end{cases}, \quad \sigma_C^* = stay, \quad \sigma_L^* = \begin{cases} down & \text{if} \quad N_H \geq \Phi_{L_a} \\ up & \text{else} \end{cases}, \tag{3.3.9}$$

with D_H^{*0} denoting the hidden trader's optimal strategy in the case without latent investors (as per proposition 7).

An immediate consequence of this result is that the hidden trader always fully exposes his order whenever he is a large investor, i.e., $N_H > \Phi_{L_a}$. This yields a fundamental difference to the baseline model. In contrast, in markets *without* upstairs interaction (i.e., segmented markets), large traders have to hide their intentions, at least partially.

Corollary 7. *In case hidden traders are large such that $N_H > \Phi_{L_a}$ holds, the optimal strategy is to fully display the trading intentions, i.e.,*

$$D_H^* = N_H. \tag{3.3.10}$$

In case hidden traders are small, i.e., $N_H < \Phi_{L_a}$, their display size is bounded, i.e.

$$D_H^* \leq \Phi_C. \tag{3.3.11}$$

Proof. Follows directly from proposition 9. □

Only sufficiently large investors, possessing the critical mass to attract latent block traders, are benefiting from total exposure. On the other hand, medium-sized or small investors, can not trigger positive liquidity externalities as large block traders require a minimum degree of liquidity exposure to trade downstairs. This observation is closely related to critical masses and liquidity externalities in fragmented markets (see, e.g., Hendershott and Mendelson (2002)).

Corollary 8 (Spread, depth, commission under latent demand)**.** *Hidden liquidity provision increases with*

(i) wider spreads,
(ii) lower opposite-side depth,
(iii) tighter liquidity competition N_C,
(iv) lower upstairs commission γ.

Proof. Follows directly from proposition 9. □

Corollary 8 derives directly from the hidden trader's equilibrium strategy D_H^* as of proposition 9. As in the benchmark case without block trading demand, the provision of hidden liquidity is affected by the liquidity competition component as discussed in corollary 5. In particular, as the cost for liquidity competition is associated with wider spreads (i), smaller opposite-side depth (ii), and competition size N_C (iii), they directly translate into a larger provision for hidden liquidity. However in addition to the benchmark case, the hidden trader propensity to provide hidden liquidity is also affected by the likelihood to attract a latent block trade counterparty. In particular, the trader will hide more when the block trader is less likely to trade downstairs. This holds, when the upstairs commission fee γ is small (see Lemma 6).

The role of the tick size is more ambiguous. While the tick size reduces liquidity competition, it reduces the block trader's incentives to trade downstairs. Both mechanisms are counterbalancing the hidden trader's incentive to expose his trading interest. However, we can identify regimes under which one of these mechanisms prevails.

Corollary 9 (Tick size)**.** *Assume $N_H < N_L - N_C$. Then there is a tick size $\Delta_0 > 0$, such that for any Δ with $\Delta > \Delta_0$, the provision in hidden liquidity increases for smaller Δ.*

Proof. Follows directly from proposition 9. □

When hidden trader's are medium-sized, i.e., $N_H < N_L - N_C$ and downstairs liquidity is not sufficient to cover the block trader's total demand, then for not too small tick sizes $\Delta > \Delta_0$, block traders will refrain from trading downstairs because of high costs. As a result, there are less benefits associated with exposure and the hidden trader adopts a camouflage strategy to avoid liquidity competition.

Since, hidden orders have a smaller chance of attracting latent trade demand, they are less likely to get executed. Consequently, hidden orders have to be cancelled more often and due to liquidation constraints, resubmitted as market orders. We have following corollary.

Corollary 10 (Dynamic implications)**.** *Large (buy) hidden orders are associated with*

(i) low fill rates (i.e. low buy execution volumes),
(ii) higher cancellation rates (i.e. increased buy cancellation volumes),
(iii) high market order re-submissions (i.e. increased buy market order volumes).

Proof. According to the model setup, the hidden trader cancels unexecuted orders and re-submits them as market orders. Hence (ii) and (iii) follow from (i). And because a larger proportion of hidden order submission reduces total displayed depth in the downstairs market, block traders are less likely to trade downstairs (i.e. Φ_{L_a} and Φ_{L_b} are high). Eventually the hidden trader faces a smaller execution probability and lower fill rates. Hence (i) follows. □

In fact, recent empirical studies tend to support this view. For instance, Bessembinder et al. (2009) suggest that hidden orders are indeed less likely to get executed and therefore more likely to get cancelled. In particular, they suggest that hidden orders are associated with *opportunity costs* which arise under timely liquidation constraints, as unfilled orders need to be cancelled eventually and (re-) submitted as *costly* market orders to meet the trade schedule. However, everything being equal, when market order sizes increase they also increasingly shift prices as orders *eat into the book*. The consequence is that hidden orders also carry a larger potential to exert price pressures and excess fluctuations.

Corollary 11 (Hidden orders and excess returns)**.** *Large (buy) hidden orders are associated with positive excess returns.*

Proof. Follows directly from corollary 10 and from the fact that market orders exert price impact according to (3.2.3). □

There is a subtle but important point about proposition 11. Strictly speaking, because they are associated with excess returns, hidden orders carry informational value. However, this predictive power does not derive from information arrival, but from *liquidity effects* or *price impact*: orders that consume liquidity on one side of the book, naturally shift prices.

Naturally, since hidden order submissions affect price returns, one also expects that there is a close link to market volatility.

Proposition 10 (Volatility)**.**

(i) ***Partial equilibrium**: Assume that the liquidity competitor and the latent block trader obey the equilibrium strategy. Moreover, suppose that the hidden trader chooses any display size D_H, not necessarily the equilibrium strategy. Then, market volatility obeys*

$$Var[R_\tau] = \beta^2\lambda^2 + \beta^2(1-q)q \begin{cases} N_H^2, & 0 \leq D_H < \Phi_{L_a} \\ (N_H - D_H - N_C)^2, & 0 < \Phi_{L_a} \leq D_H \\ (N_H - D_H)^2, & \Phi_{L_a} \leq 0. \end{cases} \tag{3.3.12}$$

(ii) ***Full equilibrium**: In equilibrium, market volatility obeys*

$$Var[R_\tau] = \beta^2\lambda^2 + \beta^2(1-q)q \begin{cases} N_H^2, & 0 \leq N_H < \Phi_{L_a} \\ N_C^2, & 0 < \Phi_{L_a} \leq N_H \\ 0, & \Phi_{L_a} \leq 0. \end{cases} \tag{3.3.13}$$

Proposition 10 is a representation of market volatility in two settings: full equilibrium and partial equilibrium. The partial equilibrium provides a more general perspective on the impact of volatility, particularly from the viewpoint of hidden liquidity provision and market transparency. By allowing the display size D_H to be arbitrary, we explicitly capture the impact of hidden liquidity provision on market volatility. In a full equilibrium, however, the dependence on the display size vanishes as it becomes a dependent variable. Furthermore, this approach allows to account for effects that can not be explained from rational behaviour alone.

As a direct consequence of the partial-equilibrium volatility (3.3.12), we infer that the amount of hidden liquidity $N_H - D_H$ affects the unconditional market volatility.

Corollary 12 (Volatility and hidden liquidity)**.** *For $N_H > N_C$, volatility increases with*

- *hidden depth, i.e. $N_H - D_H$.*

Proof. Follows directly from proposition 10 (i). □

Corollary 12 expresses the fact that large hidden orders induce price inefficiencies or trading frictions. That, is order arrivals cause temporal imbalances in the supply and demand side of liquidity. If these imbalances are not offset by additional liquidity, they materialise into price pressures as outstanding orders have to be liquidated eventually. When liquidity suppliers display their trading intentions, these imbalances can be absorbed by latent liquidity demanders. Hidden orders on the other hand, cement one-sided liquidity excesses and price pressures. In particular, this perfectly echoes the early considerations on the benefits of pre-announcements as in Admanti and Pfleiderer (1991) Grossman (2012) and Grossman and Miller (1988) and is also closely related to the *synchronization problem* in the *theory of bubbles* in Abreu and Brunnermeier (2003).

As hidden liquidity increases market volatility, so do all the factors that contribute to a larger provision in hidden liquidity (see corollary 8). In particular, market volatility is affected by the spread, the depth and the commission fee as of proposition 9.

Corollary 13 (Volatility and spread, depth, commission)**.** *Volatility increases with*

- *wider spreads,*
- *lower depth,*
- *lower (upstairs) commission fee* γ*.*

Proof. Follows directly from proposition 10 (i). □

Empirical evidence in support of a positive relationship between spread and volatility is vast and has been documented for various markets.[7] Equally, there is extensive empirical evidence on the depth-volatility link. For instance, Bessembinder and Seguin (1993) find that the relationship between depth ("open interest") and market volatility is negative for a wide range of markets, including agricultural, financial, metal and currency trading.[8]

3.4 Empirical Evidence

3.4.1 Data

To empirically test our hypotheses on the impact of hidden liquidity in a downstairs market, we require (i) information on hidden liquidity in a limit order book market and (ii) high-frequency data to model trading processes. For the former we take advantage of NASDAQ's ModelView data set. This is a unique dataset providing one-minute snapshots of the entire displayed and non-displayed depth for all NYSE-, Amex- and NASDAQ-listed stocks. Our initial dataset consists of all S&P 500 stocks that were continuously listed in the index through the period of January 2009. To reduce the impact of extreme observations, we restrict our analysis to stocks with an average daily traded volume (ADV) of less then 50 million shares, 0.2 million number of trades on average, average spreads of less than 25 cents and average price levels of less then 100 $. This reduces the sample size to $N = 452$ stocks. To reduce the impact of opening and closing auctions we restrict our analysis to daily periods between 09:15 and 15:45. Accordingly, the daily sample size for each stock consists of 390 minute-by-minute observations. Finally, we consider depth up to the best ten price levels. In addition to the NASDAQ dataset, we use

[7]For empirical evidence, see Harris (1996), Aitken et al. (2001), De Winne and D'Hondt (2009), Bollerslev and Melvin (1994), Bollerslev and Domowitz (2012), Hasbrouck and Saar (2001), Plerou et al. (2005), Wang and Yau (2000), Kalimipalli and Warga (2002).

[8]For more empirical evidence, see Ahn et al. (2001), Watanabe (2001), Ragunathan and Peker (1997), Fung and Patterson (1999).

access to the Trade and Quote Database (TAQ) to infer trade information on the full S&P 500 universe as NASDAQ does only provide insights on the quoted depth and not the amount of traded volume.

High-frequency message data is provided by the data-interface *Lobster*.[9] Lobster generates high-resolution high-frequency order book data for all NASDAQ-listed stocks. It records every order entry and provides the fully reconstructed (displayed) limit order book. The advantage of this dataset lies in its ability to facilitate the reconstruction of the full range of the order flow dynamics, from order submission to cancellation and order executions. In particular, we will be able to identify times of high-execution activity. The downside of this dataset is that it does not give a full overview of posted hidden orders. This is because electronic exchanges generally restrict transparency on their hidden orders.[10] Due to the shear size of the Lobster dataset, we restrict our analysis to a random sample of 13 stocks from the S&P 500: ABC, APC, AZO, CAH, EMR, GAS, GOOG, LEG, PAYX, SO, STJ, TDC and TROW.

By merging and combining both datasets, we obtain information about both dimensions of the order book: the full order book dynamics and the number of posted hidden orders. To merge the minute-by-minute observations of the NASDAQ Model-View data with the tick-by-tick high-frequency data set, we aggregate the order flow volumes (cancellations, submissions and executions) of the latter on a minute-by-minute basis first and then align both data sets to the right time stamps. Note that the merged data set only accounts for the thirteen stocks and not the full universe of 452 stocks that is available from NASDAQ Model-View.

Variable Definitions

We denote the total buy-side and sell-side hidden depth at time t by H_t^b and H_t^a. Equally, denote the buy-side and sell-side displayed depth by D_t^b and D_t^a. The total hidden and displayed depth are respectively denoted by H_t and D_t. The time subscript will be occasionally dropped to indicate time-averages. We define the hidden depth ratio H_t^R, i.e.

$$H_t^R := \frac{H_t}{D_t + H_t}. \tag{3.4.1}$$

The hidden and displayed order imbalances I_t^H and I_t^D are defined as

$$I_t^D := D_t^b - D_t^a, \qquad I_t^H := H_t^b - H_t^a. \tag{3.4.2}$$

Notice that a submission of a buy (sell) hidden order leads to a positive (negative) *shock* in the corresponding excess imbalance. We consider the normalised (i.e. relative) ticks ($tick$), spreads ($spread$) and return ($return$) that is absolute spread, tick and returns are divided by the prevailing mid-point price.[11] The realised variance ($vola$) is computed on a 10-minutes basis. We aggregate cancellation, submission and execution order flow for each side on a 60 seconds basis and denote them by $subbuy$ ($subsel$), $canbuy$ ($cansel$) and $exebuy$ ($exesel$).

[9]"Limit Order Book System – The Efficient Reconstruct", see http://lobster.wiwi.hu-berlin.de/.

[10]Note that executed hidden orders may be reconstructed.

[11]To ease exposition of estimation results, we refrain from using symbols but instead use shorthand words for the quantities we consider, i.e., instead of S we use $spread$ and instead of Δ we use $tick$ etc.

Descriptive Statistics

For the purpose of illustration, we recycle some basic information from chapter 1 in the figure 3.3 and the table 3.1.[12] Figure 3.3 provides a time evolution plot of total hidden H_t and displayed depth D_t for four selected stocks. Table 3.1 reports samples statistics sorted into liquidity quintiles q_i ($i \in \{1, 2, 3, 4, 5\}$). Liquidity quintiles are computed according to the average daily traded volume (ADV). Sample averages for the data from Lobster is shown in table 3.3 and autocorrelations for order flow and various order book quantities are reported in the Figure 3.5.

Several notable facts arise. First, throughout the universe, the amount of hidden trading is substantial. According to table 3.1, on average 16% of the traded and 17% of the posted volume for the S&P 500 is hidden. At the least liquid end, on average every fourth share is traded hidden. Secondly, the amount of hidden trading declines the more liquid a stock is. For instance, the ratio of traded hidden volume is almost four times larger for the least liquid quantile (26%) as compared to the most liquid quantile (7%).

Third, the fact that hidden depth exhibits a weaker serial correlation (see figure 3.5), lower mean, larger extreme values, significantly higher variations and a substantially larger right-skewness than displayed depth suggests that hidden depth is present in large volumes at few points in time. The time evolution plots of hidden and displayed in figure 3.3 depth reinforces this view: while displayed depth shows a more regular temporal pattern with lower variations, hidden liquidity concentrates around few large *spikes of liquidity*.

Fourth, order flows are highly serially correlated. In particular, the supply side of liquidity (i.e. cancellations and submissions) shows a stronger serial correlation than the demand side of liquidity (i.e. executions).

Fifth, most submitted orders get cancelled and the proportion of executed shares is low compared to overall submission volumes.

3.4.2 Cross-sectional Investigation

In this section, our primary objective is to document the validity of the cross-section implications from 3.3.2. In particular, we will address corollaries 8, 9, 12 and 13 with respect to cross-sectional effects. Therefore, we propose a simple cross-sectional regression for volatility $Vola$ and the ratio of hidden depth H^R, i.e.

$$\begin{aligned} logVola &= \alpha_v + \beta_v logH^R + \gamma_v logtick + \zeta_v logspread + \epsilon_v, && (3.4.3) \\ logH^R &= \alpha_h \qquad\qquad\quad + \gamma_h logtick + \zeta_h logspread + \epsilon_h. && (3.4.4) \end{aligned}$$

We conduct the analysis on log-transformed variables as the plain variables $tick$, $spread$, $vola$ and H^R are positive-valued. We apply standard error assumptions, i.e. iid normality with $\epsilon_v \sim N(0, \sigma_v^2)$, $\epsilon_h \sim N(0, \sigma_h^2)$ and $E[\epsilon_v \epsilon_h] = 0$. Coefficients of both models are estimated independently by ordinary least squares. We show results in table 3.2.

[12] Figure 3.3 corresponds to figure 1.1 and table 3.1 corresponds to table 1.5 of chapter 1.

Table 3.1: Cross-sectional averages for observable stock characteristics and hidden liquidity. We report averages for traded volume (ADV), the inter-arrival time of trades $time$, the trade size $size$, the spread $spread$, the mid-point price $price$ and the visible depth at the top (first level) of the book (D^{top}) and the total *posted* hidden volume H and ratio with respect to total depth D, as well as the total *traded* hidden volume H_{traded} and and its ratio with respect to total traded volume (ADV). Cross-sectional averages are grouped according to their liquidity quintiles based on ADV. On a daily basis, $HiLo$ is computed as the daily high-low difference in proportion to the prevailing daily mid-point price.

Liquidity Quantile	Observable Stock Properties							Hidden Liquidity posted		Hidden Liquidity traded	
	ADV ($10^6 sh.$)	$time$ ($sec.$)	$HiLo$ ($ratio$)	$size$ ($sh.$)	$spread$ ($ticks$)	$price$ (\$)	D^{top} ($sh.$)	H ($sh.$)	H ($ratio$)	H_{traded} ($10^6 sh.$)	H_{traded} ($ratio$)
q_1 (least)	1.39	2.65	0.07	147	4.91	36.46	308	656	0.19	0.37	0.26
q_2	2.72	1.38	0.08	158	3.39	32.84	576	1318	0.20	0.57	0.20
q_3	4.23	0.94	0.09	165	2.40	27.41	800	1671	0.17	0.69	0.15
q_4	7.13	0.61	0.10	178	1.87	24.59	1278	2292	0.16	0.83	0.11
q_5 (most)	16.98	0.35	0.11	219	1.38	23.32	3490	6202	0.13	1.10	0.07
all	6.57	1.19	0.09	174	2.79	28.91	1305	2440	0.17	0.71	0.16

The main findings are as follows: First, r^2 estimates indicate a strong goodness-of-fit. Secondly, the predictions of the corollaries 8, 9, and 12 and 13 are confirmed and statistically highly significant. T-statistics show significance at lowest conventional levels. In particular, hidden depth increases for wider spread sizes and smaller tick sizes. On the other hand, volatility is larger in markets that show a higher percentage of hidden liquidity supply.

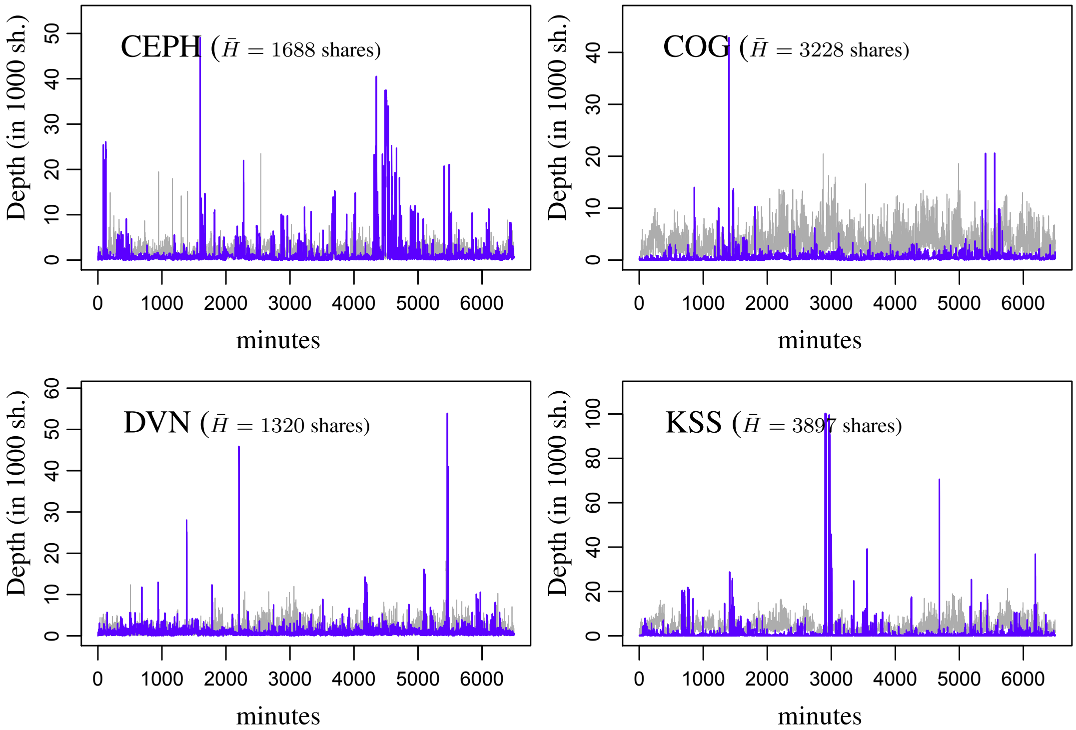
3.4.3 Dynamic (Time-Series) Investigation

Testing the impact of hidden and displayed depth on the trading process requires to properly account for multivariate dynamics. To capture the latter we suggest modelling the minute-to-minute order book dynamics using a vector autoregressive (VAR) model. VAR models for high-frequency trading and order book dynamics are initially proposed by Hasbrouck (1991) and successfully put forward by Engle and Patton (2004) and Hautsch and Huang (2012), among others. More formally, we consider the K-vector of endogenous variables y_t representing the state of the market at time t. Then y_t follows the process

$$y_t = \sum_{j=1}^{p} A_j y_{t-j} + u_t, \tag{3.4.5}$$

with A_j denoting $(K \times K)$ coefficient matrices for $j = 1, ..., p$ and u_t the vector of zero mean white noise error terms with $E\left[u_t u_t'\right] = \Sigma_u$. To account for the complexity of the limit order

Figure 3.3: Examples of minute-by-minute time evolution of total hidden and displayed depth for the period of January 2009. Evolution of total hidden depth is given in blue bars, while evolution of total displayed depth is expressed in gray colour bars. $\bar{H}$ denotes the average supply of total hidden depth.



book dynamics, including the state of the order book and order flow dynamics, we propose the following vector of endogenous variables

$$y_t := \begin{pmatrix} spread \\ I^H \\ I^D \\ D+H \\ returns \\ vola \\ exesel \\ exebuy \\ subbuy \\ subsel \\ canbuy \\ cansel \end{pmatrix}. \tag{3.4.6}$$

Quantities that refer to *quoted* depth and prices (e.g., spread, hidden and displayed imbalance, total depth, returns and volatility) are derived from NASDAQ Model-View. Quantities that refer

Table 3.2: Coefficient estimates for the cross-sectional regression of hidden liquidity, tick size, spread and volatility as of (3.4.3) and (3.4.4). H^R refers to the ratio of hidden depth relative to total depth as of (3.4.1). Numbers in brackets denote heteroscedasticity robust t-values according to White (1980).

model	$intercept$ (α)	$logH^R$	$logtick$	$logspread$	r^2
$logH^R$	-1.6258 (-14.31)	-	-0.4780 (-14.81)	0.1664 (5.09)	0.38
$logVola$	-10.5528 (-80.92)	0.3739 (7.35)	0.2989 (9.13)	0.5266 (11.00)	0.46

to order-flows instead (e.g., traded, executed and submitted orders) are inferred from the minute-by-minute aggregated Lobster dataset. Our analysis is tailored around the question how hidden and displayed liquidity shocks affect the market dynamics. In our framework, we identify buy-side (sell-side) order submissions as positive (negative) shocks to the hidden and displayed order book imbalances as defined in (3.4.2).

Using a VAR approach has the advantage of straightforwardly deriving the impact of shocks in terms of the *impulse response function* while explicitly accounting for the variables' dynamic inter-dependencies. We derive the impulse response functions from the moving-average representation of (3.4.5),

$$y_t = \Phi_0 u_t + \Phi_1 u_{t-1} + \Phi_2 u_{t-2} + \Phi_3 u_{t-3} + \dots, \tag{3.4.7}$$

with $\Phi_0 = I_K$ and $\Phi_s = \sum_{j=1}^{p} \Phi_{s-j} A_j$ for $s > 0$. We consider the *generalised impulse response* according to Pesaran and Shin (1998) that is obtained by shocking one element, while integrating out the effects of other shocks, i.e.,

$$\Delta := E\left[y_{t+n} \mid u_{jt} = \delta_j, \Omega_{t-1}\right] - E\left[y_{t+n}, \Omega_{t-1}\right] \tag{3.4.8}$$

with Ω_t denoting the information set up to t. Assuming multivariate normality for u_t, the conditional expectation given a scaled shock $\delta_j := \sqrt{\sigma_{jj}}$ in one variable leads to $E\left[u_t \mid u_{jt} = \delta_j\right] = \Sigma e_j \sigma_{jj}^{-1} \delta_j$ with e_j denoting the unit vector. Then, the generalised impulse response obeys

$$\Delta = \frac{\Phi_n \Sigma e_j}{\sqrt{\sigma_{jj}}}. \tag{3.4.9}$$

The main advantage of this approach is that the generalised impulse response functions are invariant to re-ordering of the endogenous variables. As shown by Pesaran and Shin (1998), orthogonalised impulse responses coincide with orthogonalised impulse responses (based on a Cholesky decomposition of Σ) if the respective variable is the first one in the ordering.

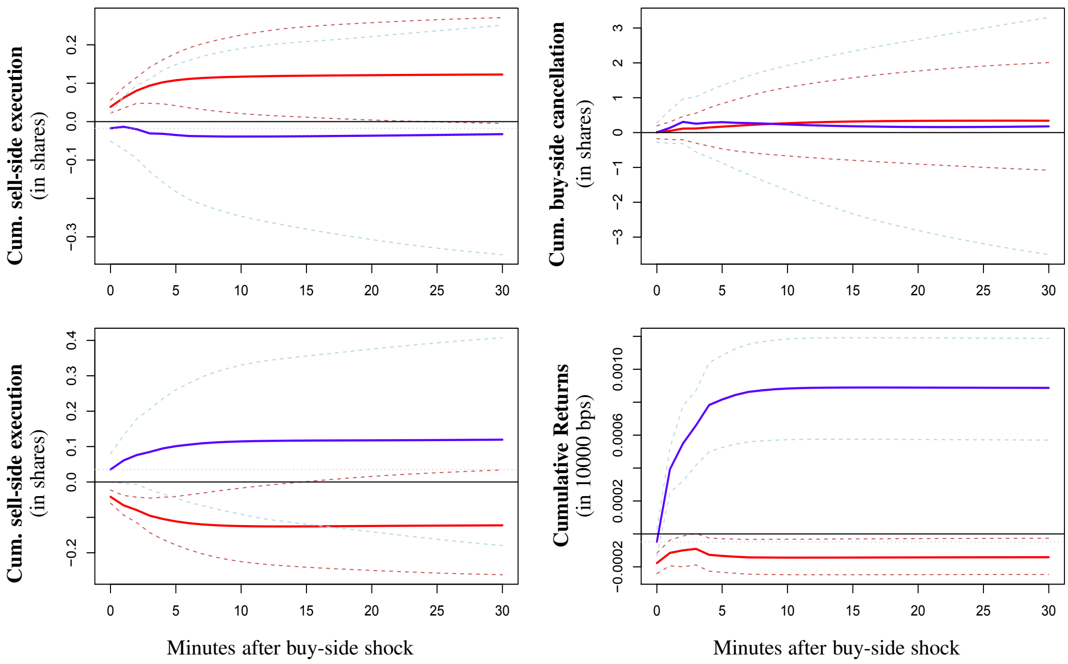
The model is applied to each stock in our high-frequency sample of thirteen stocks. In order to ease cross-sectional comparison and obtain equal lag structures in all equations, we choose a universal lag length of 3 (minutes). Based on bootstrapping using 100 replications for each

VAR estimate and impulse response, we also provide 95%-confidence intervals. For brevity of exposition, we refrain from showing the VAR estimates and impulse response estimates for the individual stocks and restrict the analysis to the cross-sectional averages. Coefficient estimates for the VAR model with the plain variables as of (3.4.6) are provided in the table 3.4 in the appendix on page 102.

Impulse Response Estimates

The cross-sectional estimates for the cumulative impulse response of executed buy shares, cancelled buy shares, submitted sell market orders and excess returns due to a buy-side (i.e. positive) shock in the hidden (solid blue curve) and displayed (solid red curve) order imbalance are shown in the figure 3.4 below.

Figure 3.4: Cross-sectional averages of estimated cumulative impulse responses of executed buy, cancelled buy and executed sell volumes and price returns due to buy-side (i.e. positive) shocks in hidden (blue) and displayed (red) order imbalances. 95% bootstrap confidence intervals shown in corresponding light colors and dashed lines.



The results confirm the predicted causal sequence as of corollaries 10 and 11. In particular, a buy-side shock in displayed depth is associated with a larger buy-side execution (ratio). Consequently, they do execute less of their orders via market orders and therefore cause less price pressures. On the other hand, as a positive (buy) shock on hidden depth does not attract counterparty demand, they are more likely to get unexecuted and cancelled. As a larger proportion of shares need to be executed via market orders, the proportional impact on price returns is larger. As a result, buy hidden orders are associated with positive excess returns.

To check for robustness, we also conduct the same analysis with the hidden and displayed imbalance variables - I_H and I_D - acting as dummy instead of plain variables. The dummy variables for imbalances are defined with value one exceeding the $p-$quantile level of total imbalances $I_T := I_H + I_D$ and zero otherwise.[13]. Our results are robust with respect to the levels $p = 70\%$ and $p = 90\%$ (see figures 3.6 and 3.7 in the appendix on page 105)

3.5 Conclusion

Hidden liquidity has become a prominent feature of todays modern stock exchanges. Despite the growing proportions of dark trading, a variety of issues remain unresolved. How hidden liquidity originates, how it interrelates with and impacts the different dimensions of the market is still an ongoing discussion. An important aspect in the issue of market transparency is related to liquidity externalities. Primary exchanges benefit when they are *thick* as order flow from latent traders migrates from anonymous trading places to the public trading place. Latent liquidity constitutes the set of possible counterparties that only trade upon pre-announcements made by, others but do not themselves issue pre-announcements of trade interests. Hence, traders can attract latent counterparties when they actively expose their interest-to-trade. The consequence is that hidden depth can pose a serious impediment to these desirable network effects.

Proposing a dynamic equilibrium framework, our work addresses these and related issues. Both, inter-market as well as intra-market competition for liquidity constitute the key building blocks of our micro-foundations. Employing a purely non-informational setting, we are able to derive several predictions. In equilibrium, large traders rather expose their trade interest in the presence of latent trade demand to increase chances of finding a counterparty. For these traders, the benefits of counterparty attraction outweigh the potential losses due to predatory trading. This is not so for medium-sized traders as they do not posses the critical mass to attract latent traders.

We predict that because wider spreads increase liquidity competition, hidden liquidity provision and volatility is larger in markets with wider spreads. This finding differs from the information based theories which predicts the opposite causation: higher volatility begets wider spreads and more hidden liquidity.

We show that hidden liquidity can increase price inefficiencies as it increases price fluctuations that are not related to fundamentals. The reason is that because hidden orders are less likely to attract counterparties, they eventually have to be traded aggressively in a less liquid market, causing higher price pressures. In fact, we show that hidden orders are related with positive excess returns.

Our theory highlights the role non-informational sourced frictions in financial markets. We show that bad market design can reduce the efficient coordination and trade mechanisms between the supply and demand side of liquidity. This is particularly so, when investors use large hidden orders as they the mismatch of mutually beneficial trades and generate price pressures.

[13]For details how imbalance threshold levels are calculated according to quantiles, see (1.4.9) in chapter 1

Appendix 3.A Proofs

Execution Volumes

Lemma 8 (Liquidity competitor's execution volume)**.** *The execution volume V_C of the liquidity competitor's limit order obeys*

$$V_C = \begin{cases} \min(x, N_C) & \textit{if} \quad (\sigma_C, \sigma_L) = (\textit{step}, \textit{up}) \\ \min((x - D_H)^+, N_C) & \textit{if} \quad (\sigma_C, \sigma_L) = (\textit{stay}, \textit{up}) \\ N_C & \textit{if} \quad \sigma_L = \textit{down} \end{cases} . \tag{3.A.1}$$

Proof. Consider the first case, i.e. $(\sigma_C, \sigma_L) = (\textit{step}, \textit{up})$. Then, because the competitor undercuts the hidden trader and submits at $B_{t_0} + \Delta$, he has price priority over the hidden trader. Hence, incoming market order shares x get first matched against the N_C of the competitor, i.e. $V_C = \min(x, N_C)$. Now assume $(\sigma_C, \sigma_L) = (\textit{stay}, \textit{up})$. This time price priority between the competitor and the hidden trader is equal. However, the displayed part - having arrived at t_0- has time priority over the competitor's order, thus $V_C = \min((x - D_H)^+, N_C)$. Finally assume $\sigma_L = \textit{down}$. Because of the block-trader's large demand, i.e. $N_L > N_H + N_C$, he will surely trade all shares from the competitor, in particular $V_C = N_C$. $\square$

Lemma 9 (Hidden trader's execution volume)**.** *The hidden trader's execution volume V_H obeys*

$$V_H = \begin{cases} \min((x - N_C)^+, N_H) & \textit{if} \quad (\sigma_C, \sigma_L) = (\textit{step}, \textit{up}) \\ \min(x, D_H) + \min((x - D_H - N_C)^+, N_H - D_H) & \textit{if} \quad (\sigma_C, \sigma_L) = (\textit{stay}, \textit{up}) \\ D_H + \min(x, N_H - D_H) & \textit{if} \quad \sigma_L = \textit{down} \end{cases} . \tag{3.A.2}$$

Proof. We can essentially recycle the arguments of the proof in Lemma 8. In case $(\sigma_C, \sigma_L) = (\textit{step}, \textit{up})$, the hidden trader has lower price priority than the competitor, thus his order gets executed only after a market order of size x has executed the competitor's N_C shares, i.e. $V_H = \min((x - N_C)^+, N_H)$. In case $(\sigma_C, \sigma_L) = (\textit{stay}, \textit{up})$, the displayed part of the hidden trader gets served first (i.e. D_H), then the competitor (i.e. N_C shares) and finally the hidden trader's hidden order (i.e. $N_H - D_H$). Thus $V_H = \min(x, D_H) + \min((x - D_H - N_C)^+, N_H - D_H)$. Finally assume $\sigma_L = \textit{down}$. Because his demand is large -i.e. $N_L > N_H + N_C$- the latent trader will trade all displayed D_H shares. The remaining $N_H - D_H$ will be traded against the noise trader. Hence, the hidden trader's execution volume reads $V_H = D_H + \min(x, N_H - D_H)$. $\square$

Payoffs

Lemma 10 (The block-trader's payoff)**.** *Given the strategies σ_L, σ_C and $\sigma_H \equiv D_H$, the block-trader's payoff Π_L obeys*

$$\Pi^L(\sigma_L, \sigma_C, \sigma_H)$$
$$= \begin{cases} -(\Delta+\gamma)\,(N_L - D_H - N_C)^+ & \textit{if} \quad (\sigma_L, \sigma_C) = (down, stay) \\ N_C\Delta - (\Delta+\gamma)\,(N_L - D_H - N_C)^+ & \textit{if} \quad (\sigma_L, \sigma_C) = (down, step) \\ -\gamma N_L & \textit{if} \quad (\sigma_L, \sigma_C) = (up, stay) \\ (\Delta - \gamma)\,N_L & \textit{if} \quad (\sigma_L, \sigma_C) = (up, step). \end{cases} \tag{3.A.3}$$

Proof. Consider the first case, i.e. $(\sigma_L, \sigma_C) = (down, stay)$. The block-trader trades all displayed depth, i.e. $D_H + N_C$ shares, at the price $B_{t_0} = 0$. Consequently, downstairs market price shifts to $B_{t_0} - \Delta$. Thus, the remaining $N_L - N_C - D_H$ shares will get executed at the upstairs price $B_{t_0} - \Delta + \gamma$ and the (relative) payoff reads $\Pi_L = -(\Delta+\gamma)\,(N_L - D_H - N_C)^+$. Now, consider the second case, i.e. $(\sigma_L, \sigma_C) = (down, step)$. In this case, everything remains the same, except the block-trader executes N_C shares at one-Δ better price. Therefore, the payoff obeys $\Pi_L = N_C\Delta - (\Delta+\gamma)\,(N_L - D_H - N_C)^+$. Consider the case $(\sigma_L, \sigma_C) = (up, stay)$. The block-trader trades all N_L shares in the upstairs market by paying a fee γ for each of the shares, thus $\Pi_L = -\gamma N_L$. Finally, assuming $(\sigma_L, \sigma_C) = (up, step)$, i.e. the block-trader again trades all N_L in the upstairs market. As the liquidity competitor improves the public best bid price, the upstairs prices shifts as well according to (3.2.5). Therefore, payoff reads $\Pi_L = (\Delta - \gamma)N_L$. □

Lemma 11 (Liquidity competitor's payoff)**.** *Given the strategies σ_L, σ_C and $\sigma_H \equiv D_H$, the liquidity competitor's payoff Π_C obeys*

$$\Pi_C(\sigma_C, \sigma_L, \sigma_H) =$$
$$\begin{cases} \Delta\min(x, N_C) + (S + \Delta + \tfrac{1}{2}\beta(N_C - x)^+)(N_C - x)^+ & if \quad (\sigma_C, \sigma_L) = (step, up) \\ \Delta N_C & if \quad (\sigma_C, \sigma_L) = (step, down) \\ \left(S + \tfrac{1}{2}\beta\left(N_C - (x - D_H)^+\right)^+\right)\left(N_C - (x - D_H)^+\right)^+ & if \quad (\sigma_C, \sigma_L) = (stay, up) \\ 0 & if \quad (\sigma_C, \sigma_L) = (stay, down). \end{cases} \tag{3.A.4}$$

Proof. Follows directly from equation (3.2.2) and Lemma (3.A.1). For instance, assume $\sigma_L = up$ and $\sigma_C = stay$, the execution volume according to Lemma 8 equals $\min((x - D_H)^+, N_C)$ shares. Thus, the payoff according to (3.2.2) reads

$$\left(S + \frac{1}{2}\beta\left(N_C - (x - D_H)^+\right)^+\right)\left(N_C - (x - D_H)^+\right)^+.$$

On the other hand, when the liquidity competitor "steps ahead", i.e. $\sigma_C = step$, then the opportunity costs associated with executing the order increases marginally by one tick Δ, i.e. the payoff reads

$$\Delta \min(x, N_C) + (S + \Delta + \frac{1}{2}\beta(N_C - x)^+)(N_C - x)^+$$

in this case. Now assume the case when the latent trader trades downstairs, i.e. $\sigma_L = down$. Then when the liquidity competitor improves the best bid (i.e. $\sigma_C = step$), his total payoff reads ΔN_C. If the competitor submits his limit order at the benchmark price $B_{t_0} = 0$ however, his execution costs are zero. □

Lemma 12 (Hidden trader's payoff)**.** *Given the strategies σ_L, σ_C and $\sigma_H \equiv D_H$, the hidden trader's payoff Π_H obeys*

$$\Pi_H(\sigma_C, \sigma_L, \sigma_H) =$$
$$\begin{cases} (S + \Delta + \frac{1}{2}\beta(N_H - (x - N_C)^+)^+)(N_H - (x - N_C)^+)^+ & if \quad (\sigma_C, \sigma_L) = (step, up) \\ (S + \frac{1}{2}\beta V_H)\,(N_H - V_H) & if \quad (\sigma_C, \sigma_L) = (stay, up) \\ (S - \Delta + \frac{1}{2}\beta(N_H - D_H - x)^+)(N_H - D_H - x)^+ & if \quad \sigma_L = down \end{cases}$$

with $V_H := \min(x, D_H) + \min((x - D_H - N_C)^+, N_H - D_H)$.

Proof. Follows directly from (3.2.2) and the execution volume V_H derived from Lemma (3.A.2). We proceed in the same fashion as before. Therefore, consider first $\sigma_L = up$ and assume $\sigma_C = step$. Because the competitor's order has priority over the hidden trader's order, in total $(x - N_C)^+$ standing (Iceberg) order shares get executed at the benchmark price $B_{t_0} = 0$. Thus remaining $(N_H - (x - N_C)^+)^+$ shares have to get executed via markets orders at the (relative) price $(S + \Delta + \frac{1}{2}\beta(N_H - (x - N_C)^+)^+)$. Consider now $\sigma_C = stay$. In this case, the execution volume reads

$$V_H = \min(x, D_H) + \min((x - D_H - N_C)^+, N_H - D_H).$$

Together with (3.2.2) one obtains the result. Finally, in the case $\sigma_L = down$, the execution volume according to Lemma 9 reads $D_H + \min(x, N_H - D_H)$. As all visible liquidity has been replenished at B_{t_0} the price shifts by a tick Δ downwards. Therefore, the remaining $N_H - D_H - \min(x, N_H - D_H) = (N_H - D_H - x)^+$ shares are executed as market orders at the price $(S - \Delta + \frac{1}{2}\beta(N_H - D_H - x)^+)$. □

Equilibrium

Proof to Lemma 5. We use Lemma 11 and the fact that $\delta = 0$ or equivalently $\sigma_L = up$. Hence, the competitor's payoff as in (3.A.4) reduces to

$$\Pi^C(\sigma_C, D_H) = \begin{cases} S\,(N_C - (x - D_H)^+)^+ & if \quad \sigma_C = stay \\ \Delta \min(x, N_C) + (S + \Delta)(N_C - x)^+ & if \quad \sigma_C = step. \end{cases} \tag{3.A.5}$$

We want to find the strategy σ_C^* that minimizes the competitor's expected payoff given the hidden trader choses to display D_H shares, i.e.

$$\sigma_C^* \equiv \arg\min_{\sigma_C \in \Sigma_C} \mathbb{E}[\Pi_C(\sigma_C, D_H)].$$

From (3.A.5), we infer that $\mathbb{E}[\Pi_C(\sigma_C = stay, D_H = 0)] < \mathbb{E}[\Pi_C(\sigma_C = step, D_H = 0)]$ holds. Thus because of continuity, for sufficiently small display sizes D_H, $\sigma_C = stay$ is the optimal strategy for the competitor. On the other hand, $\mathbb{E}[\Pi_C(\sigma_C = stay, D_H)]$ is monotonously increasing in the display size D_H, whereas it is constant for $\sigma_C = step$. Lets denote Φ_C the critical threshold when both strategies exactly trade-off (If no such finite threshold exists, we write $\Phi_C = \infty$). Then the optimal strategy can be expressed in the following way

$$\sigma_C^* = \begin{cases} stay & \text{if} \quad D_H \leq \Phi_C \\ step & \text{else} \end{cases}.$$

We obtain Φ_C by simply equating both payoffs $\mathbb{E}[\Pi_C(\sigma_C = stay, D_H)]$ and $\mathbb{E}[\Pi_C(\sigma_C = step, D_H)]$ and solving for $D_H \equiv \Phi_C$. That is

$$\begin{aligned} 0 &= \mathbb{E}[\Pi_C(\sigma_C = stay, \Phi_C)] - \mathbb{E}[\Pi_C(\sigma_C = step, \Phi_C)] \\ &= \left(\left(1 - e^{-\frac{N_C}{\lambda}}\right)\left(1 - e^{-\frac{\Phi_C}{\lambda}}\right)\lambda(\beta\lambda - S) + N_C e^{-\frac{\Phi_C}{\lambda}}\left(\beta\lambda + e^{\frac{\Phi_C}{\lambda}}(\Delta - \beta\lambda)\right)\right). \end{aligned}$$

Solving for Φ_C, we can finally rewrite the latter expression as

$$\Phi_C = \begin{cases} \lambda \log\left(\frac{1}{1-g}\right) & \text{if} \quad g < 1 \\ \infty & \text{else} \end{cases}$$

with

$$g := \frac{N_C}{\lambda} \frac{\Delta}{S\left(1 - e^{-\frac{N_C}{\lambda}}\right) + \beta\left(N_C - \lambda\left(1 - e^{-\frac{N_C}{\lambda}}\right)\right)}.$$

□

Proof of Proposition 7. Because of Lemma 5 and the fact that $\lambda, N_C, \Delta > 0$ holds, the display threshold is positive, i.e. $\Phi_C > 0$. Let us therefore consider the first case, i.e. $N_H < \Phi_C$. Then because of $D_H \leq N_H$ and by Lemma 5, the liquidity competitor *stays* at the same price level as the hidden trader, i.e. $\sigma_C^* = stay$. Thus according to Lemma 12, the hidden trader's (expected) payoff reads

$$\begin{aligned} \mathbb{E}\Big[\Pi_H(\sigma_C = stay, \sigma_H = D_H)\Big] &= \mathbb{E}\Big[(S + \frac{1}{2}\beta(N_H - V_H))(N_H - V_H)\Big] \\ &= SN_H + \frac{1}{2}\beta N_H^2 - \mathbb{E}[V_H](S + \beta N_H) + \frac{1}{2}\beta\mathbb{E}[(V_H)^2]. \end{aligned}$$

By Corollary 12, the hidden trader's payoff is monotonously decreasing in the display size D_H. Hence, $D_H^* = N_H$ and $\sigma_C^* = stay$. Now assume the opposite case, i.e. $N_H \geq \Phi_C$ holds. Following the same reasoning, in case $D_H \leq \Phi_C$, the competitor chooses the $stay$-strategy and therefore we have

$$\mathbb{E}[\Pi_H(\sigma_H = D_H)] \geq \mathbb{E}[\Pi_H(\sigma_H = \Phi_C)], \quad D_H \leq \Phi_C.$$

Hence, $D_H^* \geq \Phi_C$. It remains to be shown that $D_H^* \leq \Phi_C$ holds. Therefore, consider the following expression

$$\begin{aligned}
&\mathbb{E}[\Pi_H(D_H \leq \Phi_C) - \mathbb{E}[\Pi_H(\sigma_H > \Phi_C)] = \\
&= \mathbb{E}\Big[(S + \frac{1}{2}\beta(N_H - V_H))(N_H - V_H)\Big|\sigma_H \leq \Phi_C\Big] \\
&\qquad\qquad - \mathbb{E}\Big[(S + \Delta + \frac{1}{2}\beta(N_H - V_H))(N_H - V_H)\Big|\sigma_H > \Phi_C\Big] \\
&\overset{(*)}{=} -\Delta \underbrace{(N_H - \mathbb{E}[V_H|\sigma_H > \Phi_C])}_{\geq 0} + (S + \beta N_H)\underbrace{(\mathbb{E}[V_H|\sigma_H > \Phi_C] - \mathbb{E}[V_H|\sigma_H \leq \Phi_C])}_{<0} \\
&\qquad\qquad + \frac{1}{2}\beta\underbrace{\Big(\mathbb{E}[(V_H)^2|\sigma_H \leq \Phi_C] - \mathbb{E}[(V_H)^2|\sigma_H > \Phi_C]\Big)}_{<0} \\
&< 0.
\end{aligned}$$

The negativity of the first term in $(*)$ follows because $N_H \geq V_H$ by definition. The signs of the second and third terms follow directly from Lemma 9 and the fact that in equilibrium the competitor choses $\sigma_C = step$ in case $\sigma_H > \Phi_C$ and $\sigma_C = stay$ otherweise. Thus finally, $D_H^* \leq \Phi_C$ and therefore $D_H^* = \Phi_C$. □

Proof of Lemma 6 (Block-trader's best response). First assume $\sigma_C = stay$. Then according to the block investor's payoff (3.A.3), the definition of Φ_{L_a} and the case $0 \leq D_H \leq \Phi_{L_a}$, we have

$$\begin{aligned}
\Pi_L(\sigma_L = up, \sigma_C = stay) - \Pi_L(\sigma_L = down, \sigma_C = stay) &= \\
&= -\gamma N_L + (\Delta + \gamma)(N_L - D_H - N_C)^+ \\
&\geq -\gamma N_L + (\Delta + \gamma)(N_L - \Phi_{L_a} - N_C)^+ \\
&= 0.
\end{aligned}$$

Thus, for $D_H \leq \Phi_{L_a}$ and $\sigma_C = stay$, the block-trader's optimal strategy obeys $\sigma_L^* = down$. In a total analogous way, we obtain for $\Phi_{L_a} < D_H$, $\sigma_L^* = up$ and we arrive at

$$\sigma_L^* = \begin{cases} down & \text{for} \quad D_H \leq \Phi_{L_a} \\ up & \text{else} \end{cases} \qquad \text{for} \quad \sigma_C = stay. \tag{3.A.6}$$

Now we consider the case $\sigma_C = step$. We proceed in the same fashion, according to (3.A.3) and for $0 \leq D_H \leq \Phi_{L_b}$, we have

$$\begin{aligned}
\Pi_L(\sigma_L = up, \sigma_C = step) - \Pi_L(\sigma_L = down, \sigma_C = step) = \\
&= (\Delta - \gamma)N_L - N_C\Delta + (\Delta + \gamma)(N_L - D_H - N_C)^+ \\
&\geq (\Delta - \gamma)N_L - N_C\Delta + (\Delta + \gamma)(N_L - \Phi_{L_b} - N_C)^+ \\
&= (\Delta - \gamma)N_L - \Delta N_C + N_L(\gamma - \Delta) + N_C\Delta \\
&= 0.
\end{aligned}$$

In other words, for $D_H \leq \Phi_{L_b}$ and $\sigma_C = step$, the block-trader's optimal strategy is $\sigma_L^* = down$. In a total analogous way, we obtain for $\Phi_{L_b} < D_H$, that the block-trader's optimal strategy is $\sigma_L^* = up$. We thus have

$$\sigma_L^* = \begin{cases} down & \text{for} \quad D_H \leq \Phi_{L_b} \\ up & \text{else} \end{cases} \qquad \text{for} \quad \sigma_C = step. \tag{3.A.7}$$

Because of $N_L > N_C$, we have $\Phi_{L_a} < \Phi_{L_b}$ and we can finally sum up both results (3.A.6) and (3.A.7)

$$\sigma_L^* = \begin{cases} up & \text{if} \quad 0 \leq D_H < \Phi_{L_a} \\ down & \text{if} \quad \Phi_{L_a} \leq D_H < \Phi_{L_b} \quad \text{and} \quad \sigma_C = stay \\ up & \text{if} \quad \Phi_{L_a} \leq D_H < \Phi_{L_b} \quad \text{and} \quad \sigma_C = step \\ down & \text{if} \quad \Phi_{L_b} \leq D_H \end{cases}.$$

□

Proof to Lemma 7 (Liquidity Competitor's Best Response with Latent Investor). First assume that $D_H \leq \Phi_{L_a}$. Then because of lemma 6, the latent trader will never trade downstairs i.e. $\sigma_L = up$. For the liquidity competitor and the hidden trader, this problem effectively reduces to the case without latent trader. We can thus recycle the results of Proposition 7

$$\sigma_C^* = \sigma_C^{*0} \qquad \text{for} \qquad 0 \leq D_H < \Phi_{L_a},$$

σ_C^{*0} referring to the competitor's eq. strategy without latent demand (i.e. $\delta = 0$).

Now assume $\Phi_{L_a} < D_H \leq \Phi_{L_b}$. According to Lemma 6, the latent trader trades downstairs if (and only if) the competitor does not improve the best bid price, i.e. if $\sigma_C = stay$ holds. However, according to Lemma 10, $\Pi_C(\sigma_L = up, \sigma_C = step) - \Pi_C(\sigma_L = down, \sigma_C = stay) > 0$ holds for any $x \geq 0$. Thus

$$\sigma_C^* = stay \qquad \text{for} \qquad \Phi_{L_a} < D_H \leq \Phi_{L_b}.$$

Finally, consider the case $\Phi_{L_b} < D_H$. Again using Lemma 6, the latent trader will trade downstairs, i.e. $\sigma_L = down$ and the payoff according to Lemma 10 obey

$$\Pi_C(\sigma_L = down, \sigma_C = stay) - \Pi_C(\sigma_L = down, \sigma_C = step) = -\Delta N_C < 0 \qquad x \geq 0.$$

If the latter inequality holds for all x, so it holds also in expectation. Thus in this case, the liquidity competitor's optimal strategy is

$$\sigma_C^* = stay \qquad \text{if} \quad D_H > \Phi_{L_a} \quad \text{holds.}$$

$\square$

Proof to Proposition 9 (Equilibrium with block investors). The best response strategies of the liq. competitor and the lat. trader have been shown in Lemma 6 and 7. To derive the equilibrium, the hidden trader's optimal strategy remains to be shown. For that end assume $N_H \leq \Phi_{L_a}$. Because $D_H \leq N_H \leq \Phi_{L_a}$ and because of lemma 6, the latent trader will never trade downstairs, i.e. $\sigma_L = up$. Hence, the hidden trader's game reduces to the problem with the latent investor and therefore

$$\sigma_H^* = \sigma_H^{*0} \qquad\qquad \text{for} \qquad\qquad N_H \leq \Phi_{L_a},$$

σ_H^{*0} referring to the hidden trader's eq. strategy without latent investor.

Now assume the opposite, i.e. $N_H > \Phi_{L_a}$. Because of the previous Lemma 7 and Lemma 6, in case $D_H > \Phi_{L_a}$, the competitor will go for $\sigma_C = stay$ and the latent investor for $\sigma_L = down$. Hence, the payoff in this case according to Lemma reads

$$\begin{aligned}
&\mathbb{E}[\Pi_H|D_H > \Phi_{L_a}] \\
=&\mathbb{E}[\Pi_H|\sigma_L = down, \sigma_C = stay, D_H > \Phi_{L_a}] \\
=&\mathbb{E}[(S + \frac{1}{2}\beta(N_H - V_H))(N_H - V_H)|\sigma_L = down, \sigma_C = stay, D_H > \Phi_{L_a}] \\
\geq&\mathbb{E}[(S + \frac{1}{2}\beta(N_H - V_H))\underbrace{(N_H - V_H)}_{=0}|\sigma_L = down, \sigma_C = stay, D_H = N_H] \\
=&\mathbb{E}[\Pi_H|D_H = N_H] \\
=&0.
\end{aligned}$$

On the other hand,

$$\begin{aligned}
\mathbb{E}[\Pi_H|D_H \leq \Phi_{L_a}] \geq&\mathbb{E}[\Pi_H|D_H \leq \Phi_{L_a}, \sigma_C = stay] \\
=&\mathbb{E}[(S + \frac{1}{2}\beta(N_H - V_H))\,(N_H - V_H)\,|D_H \leq \Phi_{L_a}, \sigma_C = stay] \\
\geq&S\mathbb{E}[(N_H - V_H)\,|D_H \leq \Phi_{L_a}, \sigma_C = stay] \\
\geq&S\mathbb{E}[(N_H - V_H)\,|D_H \leq \Phi_{L_a}, \sigma_C = stay, N_C = 0] \\
=&S\left(N_H - \lambda(1 - e^{-\frac{N_H}{\lambda}})\right) \\
>&0
\end{aligned}$$

for finite $\lambda > 0$. Thus $D_H^* = N_H$. $\square$

Volatility

Proofs to Proposition 8 (Equilibrium volatility without latent block-traders). The proof is identical to the case $\Phi_L > \Delta$ of Proposition 10, i.e. see equation (3.A.8). $\square$

Proofs to Proposition 10 (Partial equilibrium volatility with latent block-traders). To calculate the partial equilibrium volatility, we consider that both, the latent block trader observes his equilibrium strategy and that the liquidity competitor observes the fix strategiy $\sigma_C = stay$ which is the action he would take if the game is in equilibrium. However, we allow the Iceberg trader to choose an arbirtrary display size Δ. We first construct the midquote return. Therefore, denote the total sell market order volume at t_2 by $\bar{X}$. The sell market order volume consists on both the demand from the noise trader and the demand from the latent block trader, i.e. $\bar{X} = x + \bar{x_L}$.

$$\bar{x_L} = \begin{cases} \bar{\Delta} + C & \text{if} \quad \sigma_L = down \\ 0 & \text{if} \quad \sigma_L \neq down. \end{cases}$$

$\bar{\Delta}$ denotes the observed display quantity, and $\bar{N_H}$ the total to be traded shares of the iceberg trader. Both quantities take on the values Δ and N_H when the iceberg trader arrives. His arrival probability is p. The mean and variances of binomial random variables obey

$$\mathbb{E}[\bar{N_H}] = pN_H, \qquad \mathbb{E}[\bar{\Delta}] = p\Delta,$$
$$Var[\bar{N_H}] = p(1-p)N_H^2, \qquad Var[\bar{\Delta}] = p(1-p)\Delta^2.$$

Both, $\bar{N_H}$ and $\bar{\Delta}$, are correlated with covariance

$$\begin{aligned} Cov[\bar{N_H}, \bar{\Delta}] &= \mathbb{E}[\bar{N_H}\bar{\Delta}] - \mathbb{E}[\bar{N_H}]\mathbb{E}[\bar{\Delta}] \\ &= p(1-p)N_H\Delta. \end{aligned}$$

By assumption, the hidden trader's arrival is independent of the noise trader demand x, thus

$$Cov[x, \bar{x_L}] = 0, \qquad Cov[x, \bar{\Delta}] = 0.$$

Because, we assume constant spreads, i.e. $a_t = b_t + s$ and linear price impact, excess buy volume, i.e. $H + C > \bar{X}$ materializes in positive linear shifts, while excess sell volume $H + C \leq \bar{X}$ materializes in negative shifts. Hence, prices at terminal time read

$$a_{t+\tau} = a_t + \beta(\bar{N_H} + C - \bar{X}), \qquad b_{t+\tau} = b_t + \beta(\bar{N_H} + C - \bar{X}).$$

With the midquote price, i.e. $p_t^{mid} = \frac{a_t + b_t}{2}$, its return reads

$$R_\tau = p_{t+\tau}^{mid} - p_t^{mid} = \beta(\bar{N_H} + C - \bar{X}).$$

We first compute the variance for the case $\Phi_L > \Delta \geq 0$. In this case $\sigma_L \neq down$ holds, i.e. $\bar{x_L} = 0$ and $\bar{X} = x$.

$$\begin{aligned} Var[R_\tau] &= Var[\beta(\bar{N_H} + C - x)] \\ &= \beta^2 Var[(\bar{N_H} - x)] \\ &= \beta^2 \underbrace{Var[x]}_{\lambda^2} - \beta^2 \underbrace{Cov[x, \bar{N_H}]}_{=0} + \beta^2 \underbrace{Var[N_H]}_{=p(1-p)N_H} \\ &= \beta^2\lambda^2 + p(1-p)\beta^2 N_H^2. \end{aligned}$$

In the second equation we used the fact that C is deterministic and fix. Now we consider the case $0 < \Phi_L \leq \Delta$. In this case $\sigma_L = down$, i.e. $\bar{X} = x + \bar{x_L}$. Note that in this case $\bar{x_L}$ it itself binomial.

$$
\begin{aligned}
Var[R_\tau] &= Var[\beta(\bar{N_H} + C - \bar{X})] \\
&= \beta^2 Var[\beta(\bar{N_H} - x - \bar{x_L})] \\
&= \beta^2 \underbrace{Var[\bar{N_H}]}_{=p(1-p)N_H} + \beta^2 \underbrace{Var[x]}_{=\lambda^2} + \beta^2 \underbrace{Var[\bar{x_L}]}_{=p(1-p)(\Delta+C)^2} \\
&\qquad - 2\beta^2 \underbrace{Cov[\bar{N_H}, x]}_{=0} - 2\beta^2 Cov[\bar{N_H}, \bar{x_L}] + 2\beta^2 \underbrace{Cov[x, \bar{x_L}]}_{=0} \\
&= p(1-p)\beta^2 N_H^2 + \beta^2\lambda^2 + p(1-p)\beta^2(\Delta + C)^2 - 2\beta^2 \underbrace{Cov[\bar{N_H}, \bar{x_L}]}_{=p(1-p)N_H(\Delta+C)} \\
&= \beta^2\lambda^2 + p(1-p)\beta^2\Big(N_H^2 - 2N_H(\Delta + C) + (\Delta + C)^2\Big) \\
&= \beta^2\lambda^2 + p(1-p)\beta^2\Big(N_H - \Delta - C\Big)^2.
\end{aligned}
\tag{3.A.8}
$$

Least, we consider the case $\Phi_L \leq 0 \leq \Delta$.

$$
\begin{aligned}
Var[R_\tau] &= Var[\beta(\bar{N_H} + C - \bar{X})] \\
&= \beta^2 Var[\beta(\bar{N_H} - x - \bar{x_L})] \\
&= \beta^2 Var[\beta(\bar{N_H} - x - C - \hat{\Delta})] \\
&= \beta^2 Var[\beta(\bar{N_H} - x - \hat{\Delta})] \\
&= \beta^2 \underbrace{Var[\bar{N_H}]}_{=p(1-p)N_H} + \beta^2 \underbrace{Var[x]}_{=\lambda^2} + \beta^2 \underbrace{Var[\bar{\Delta}]}_{=p(1-p)\Delta^2} \\
&\qquad - 2\beta^2 \underbrace{Cov[\bar{N_H}, x]}_{=0} - 2\beta^2 Cov[\bar{N_H}, \bar{\Delta}] + 2\beta^2 \underbrace{Cov[x, \bar{\Delta}]}_{=0} \\
&= p(1-p)\beta^2 N_H^2 + \beta^2\lambda^2 + p(1-p)\beta^2\Delta^2 - 2\beta^2 \underbrace{Cov[\bar{N_H}, \bar{\Delta}]}_{=p(1-p)N_H\Delta} \\
&= \beta^2\lambda^2 + p(1-p)\beta^2\Big(N_H^2 - 2N_H\Delta + \Delta^2\Big) \\
&= \beta^2\lambda^2 + p(1-p)\beta^2\Big(N_H - \Delta\Big)^2.
\end{aligned}
$$

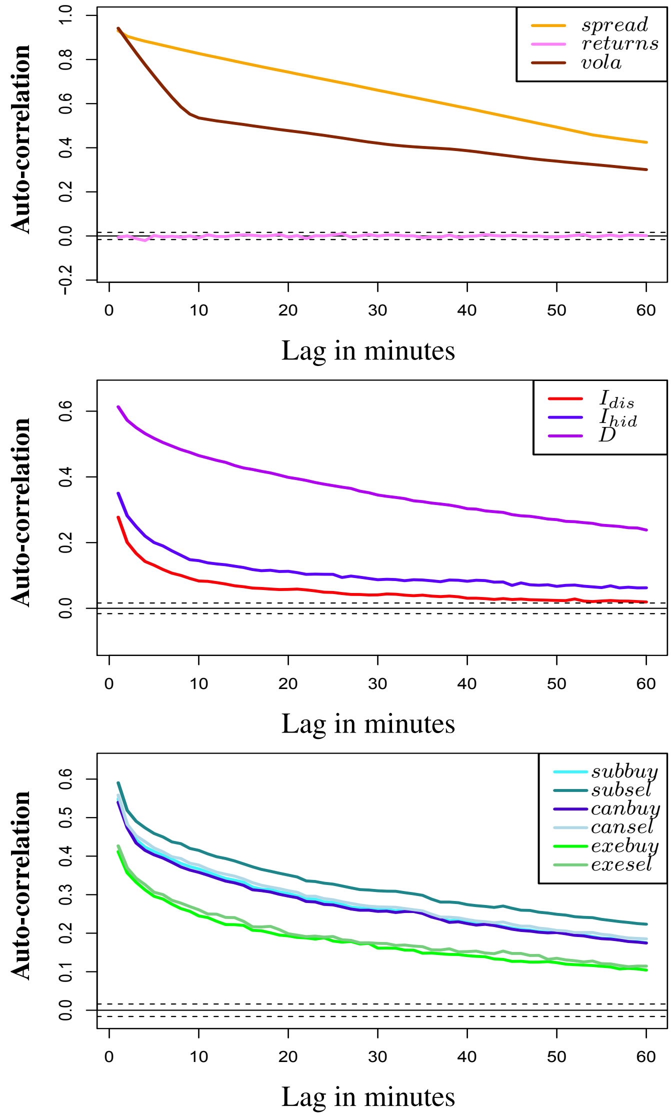
$\square$

Appendix 3.B Descriptive Statistics

Table 3.3: Time averages for order book (midpoint price, spread and first-level depth) and order flow variables (order submissions, cancellations and executions) for fourteen NASDAQ stocks. Standard deviations are given in round brackets. Order flow averages are rounded to the nearest integer, while order book variables are rounded to the second digit.

	Order Book Variables			Order Flow Variables					
stock	*price* ($)	*spread* (*tick*)	*depth* (*shares*)	*subbuy* (*shares*)	*subsel* (*shares*)	*canbuy* (*shares*)	*cansel* (*shares*)	*exebuy* (*shares*)	*exesel* (*shares*)
ABC	33.08	7.36	303.13	6542	5556	6799	6586	451	456
	(2.9)	(55.32)	(287.1)	(8005)	(11342)	(8137)	(11440)	(702)	(724)
APC	36.8	4.92	286.29	24432	22936	23755	23319	1822	1830
	(3.68)	(11.1)	(287.09)	(32158)	(24566)	(31640)	(23818)	(1978)	(1946)
AYE	29.4	5.16	323.89	9427	8157	9757	9783	596	566
	(4.15)	(17)	(281.96)	(11425)	(16205)	(11471)	(16394)	(888)	(829)
AZO	129.2	31.98	167.35	6051	3493	5830	5881	277	272
	(18.76)	(172.4)	(207)	(17634)	(10668)	(17581)	(18701)	(493)	(466)
CAH	35.65	4.62	300.3	7847	6886	8129	8075	541	567
	(4.25)	(12.1)	(300.32)	(14021)	(16196)	(14049)	(16088)	(836)	(954)
EMR	32.26	3.17	458.38	23349	22689	22973	23450	1927	1636
	(3.34)	(10.27)	(455.57)	(22090)	(26040)	(21810)	(26302)	(1919)	(1843)
GAS	36.23	13.09	197.27	4737	4637	4765	4958	154	168
	(4.84)	(82.44)	(148.12)	(11012)	(13715)	(10968)	(13561)	(325)	(390)
GOOG	326.93	28.7	189.43	14369	9958	12039	12347	2170	2143
	(29.75)	(18.18)	(382.8)	(44827)	(29756)	(44288)	(30281)	(2481)	(2453)
LEG	14.34	2.28	788.68	13759	12091	13731	13703	718	669
	(2.27)	(7.22)	(650.43)	(30221)	(23755)	(29994)	(27180)	(1039)	(1022)
PAYX	25.55	1.83	1281.12	35648	34574	34796	35503	2401	2391
	(2.19)	(4.72)	(1374.66)	(30621)	(36847)	(29758)	(35713)	(3488)	(3393)
SO	33.68	2.17	788.97	21601	20903	20914	21454	1784	1763
	(2.75)	(5.38)	(792.87)	(22905)	(24071)	(22158)	(24796)	(2195)	(2167)
STJ	34.21	4.78	332.59	13304	11926	13188	12608	951	946
	(3.5)	(26.16)	(288.1)	(20476)	(17179)	(20170)	(16854)	(1251)	(1263)
TDC	14.88	2.8	441.27	5751	4973	5977	6050	342	350
	(1.43)	(6.17)	(413.98)	(6388)	(5597)	(6549)	(6413)	(606)	(656)
TROW	31.58	6.21	384.1	19763	20066	18676	18883	1767	1800
	(6.69)	(25.98)	(528.23)	(22421)	(23890)	(21658)	(23141)	(2935)	(3295)

Figure 3.5: Sample Autocorrelations for orderflow (cancellations, submissions, executions etc.) and order book quantities (hidden imbalance, displayed imbalance, total depth, volatility, returns and spread).



Appendix 3.C VAR Estimates

Table 3.4: Cross-sectional averages of coefficient estimates of the VAR(3) model in (3.4.5). The endogenous state vector y_t consists of order book and order flow variables, see (3.4.6). T-statistics are reported in round brackes. "**", "*" and "·" denote significance at the 0.1%, 1% and 10% level respectively.

	Order Book Variables						Order Flow Variables					
Variable		Imbalances					Submissions		Cancellations		Executions	
$lag = 1\text{min}$	$spread$	I^D	I^H	$(D+H)$	$returns$	$vola$	$subbuy$	$subsel$	$canbuy$	$cansel$	$exebuy$	$exesel$
$spread_{t-1}$	$6.5e-01^{**}$	$1.9e-02$	$6.9e-02$	$-1.4e-02^{**}$	$1.2e+01$	$2.2e-02^{\cdot}$	$-2.7e-02$	$-1.7e-02$	$-2.7e-02$	$-1.9e-02$	$-3.9e-02^{\cdot}$	$-3.7e-02^{\cdot}$
$(t-stat)$	$(7.9e+01)$	$(-1.6e-01)$	$(2.4e-01)$	$(-6.3e+00)$	$(1.0e+00)$	$(7.4e+00)$	$(-3.1e+00)$	$(-2.2e+00)$	$(-3.2e+00)$	$(-2.5e+00)$	$(-2.8e+00)$	$(-2.6e+00)$
I^D_{t-1}	$4.8e-04$	$2.2e-01^{**}$	$4.2e-02$	$-1.7e-04$	$6.9e-02$	$-2.2e-05$	$-4.7e-04$	$-4.0e-04$	$-3.1e-04$	$-2.6e-04$	$8.4e-06$	$-2.4e-03$
$(t-stat)$	$(5.2e-01)$	$(2.6e+01)$	$(5.9e-01)$	$(-7.4e-02)$	$(-1.8e+00)$	$(-1.7e-01)$	$(-3.1e-01)$	$(-2.7e-01)$	$(-1.2e-01)$	$(-4.3e-02)$	$(4.8e-01)$	$(-1.2e+00)$
I^H_{t-1}	$1.9e-04$	$-3.4e-05$	$2.6e-01^{**}$	$-5.6e-05$	$7.4e-01^{**}$	$1.0e-04$	$7.7e-05$	$-2.4e-04$	$3.9e-05$	$-1.2e-04$	$-3.5e-04$	$9.3e-05$
$(t-stat)$	$(1.8e-01)$	$(-1.8e-01)$	$(3.1e+01)$	$(4.6e-02)$	$(-1.1e+00)$	$(3.8e-01)$	$(2.0e-02)$	$(-1.3e-01)$	$(1.8e-01)$	$(-1.5e-01)$	$(-2.1e-01)$	$(-1.7e-01)$
$(D+H)_{t-1}$	$-5.2e-02$	$3.0e-01$	$4.7e+00$	$3.3e-01^{**}$	$-5.5e+01$	$-2.2e-02$	$3.1e-02$	$4.7e-02$	$4.7e-02$	$5.1e-02$	$1.1e-01$	$1.2e-01$
$(t-stat)$	$(-2.5e+00)$	$(4.8e-01)$	$(1.2e+00)$	$(4.1e+01)$	$(-1.1e+00)$	$(-2.0e+00)$	$(1.1e+00)$	$(1.7e+00)$	$(1.7e+00)$	$(2.0e+00)$	$(2.8e+00)$	$(2.9e+00)$
$returns_{t-1}$	$-4.0e-06$	$-3.7e-04$	$-2.4e-03$	$-5.2e-06$	$-2.7e-02$	$2.7e-06$	$2.3e-05$	$-2.2e-05$	$1.6e-05$	$-1.4e-06$	$-3.5e-05$	$2.8e-05$
$(t-stat)$	$(-3.7e-01)$	$(-9.1e-01)$	$(-5.0e-01)$	$(-5.1e-01)$	$(-3.2e+00)$	$(3.0e-02)$	$(8.0e-01)$	$(-1.1e+00)$	$(5.9e-01)$	$(1.6e-01)$	$(-4.2e-01)$	$(2.7e-01)$
$vola_{t-1}$	$-9.3e-02$	$1.4e-01$	$2.6e-01$	$-2.6e-02^{*}$	$1.3e+01$	$9.4e-01^{**}$	$-1.7e-02$	$-1.1e-03$	$-1.9e-02$	$-3.5e-03$	$-6.0e-02$	$-7.7e-02$
$(t-stat)$	$(-4.1e+00)$	$(3.2e-01)$	$(-1.0e-01)$	$(-4.2e+00)$	$(4.2e-01)$	$(1.1e+02)$	$(-1.0e+00)$	$(-4.9e-01)$	$(-1.2e+00)$	$(-6.3e-01)$	$(-1.8e+00)$	$(-2.3e+00)$
$subbuy_{t-1}$	$-1.3e-01^{*}$	$-1.8e+00^{\cdot}$	$-5.5e+00$	$2.1e-02^{\cdot}$	$-8.6e+02^{**}$	$-1.2e-01^{**}$	$2.3e-01^{**}$	$1.8e-01^{*}$	$1.2e-01$	$1.1e-01^{\cdot}$	$4.0e-01^{**}$	$1.1e-01$
$(t-stat)$	$(-4.2e+00)$	$(-4.2e-01)$	$(1.0e+00)$	$(2.3e+00)$	$(-1.7e+01)$	$(-9.6e+00)$	$(8.5e+00)$	$(6.8e+00)$	$(4.9e+00)$	$(4.3e+00)$	$(8.1e+00)$	$(2.1e+00)$
$subsel_{t-1}$	$-1.2e-01^{*}$	$6.1e-01$	$-1.2e-01$	$3.1e-02$	$-2.3e+02^{**}$	$-8.5e-02^{**}$	$1.2e-01$	$3.3e-01^{*}$	$1.0e-01$	$7.5e-02^{*}$	$8.6e-02$	$2.4e-01^{**}$
$(t-stat)$	$(-4.9e+00)$	$(-8.0e-01)$	$(-4.8e-01)$	$(3.2e+00)$	$(-1.7e+00)$	$(-7.8e+00)$	$(4.6e+00)$	$(1.8e+01)$	$(4.7e+00)$	$(4.8e+00)$	$(1.8e+00)$	$(5.6e+00)$
$canbuy_{t-1}$	$1.6e-01^{\cdot}$	$1.8e+00$	$2.5e+00$	$-2.0e-02$	$9.4e+02^{**}$	$1.3e-01^{**}$	$5.1e-02$	$-9.5e-02^{*}$	$1.6e-01^{*}$	$-1.2e-02$	$-3.3e-01^{**}$	$-6.9e-02$
$(t-stat)$	$(5.1e+00)$	$(7.1e-01)$	$(-1.3e+00)$	$(-2.1e+00)$	$(1.8e+01)$	$(1.1e+01)$	$(2.1e+00)$	$(-4.1e+00)$	$(6.0e+00)$	$(-1.1e+00)$	$(-7.0e+00)$	$(-1.4e+00)$
$cansel_{t-1}$	$9.1e-02^{*}$	$-5.8e-01$	$2.1e+00$	$-2.9e-02$	$1.7e+02^{**}$	$1.1e-01^{**}$	$9.9e-03$	$-1.2e-02$	$3.1e-02$	$2.3e-01^{**}$	$-6.2e-02$	$-1.9e-01^{*}$
$(t-stat)$	$(3.5e+00)$	$(5.3e-01)$	$(6.0e-01)$	$(-2.7e+00)$	$(9.2e-01)$	$(1.0e+01)$	$(6.9e-01)$	$(-1.3e+00)$	$(1.2e+00)$	$(1.1e+01)$	$(-1.1e+00)$	$(-4.2e+00)$
$exebuy_{t-1}$	$2.2e-02^{*}$	$1.7e-02$	$5.6e-01$	$-3.6e-03$	$7.8e+01^{**}$	$2.3e-02^{**}$	$1.8e-02^{\cdot}$	$-2.2e-03$	$9.3e-03$	$1.5e-03$	$2.0e-01^{**}$	$8.4e-02^{**}$
$(t-stat)$	$(4.3e+00)$	$(-1.3e+00)$	$(-1.2e+00)$	$(-2.3e+00)$	$(8.6e+00)$	$(9.4e+00)$	$(2.6e+00)$	$(-6.7e-01)$	$(1.2e+00)$	$(7.2e-02)$	$(2.1e+01)$	$(8.9e+00)$
$exesel_{t-1}$	$2.8e-02$	$-6.9e-02$	$5.4e-02$	$-5.4e-03$	$-3.3e+01^{**}$	$2.5e-02^{**}$	$-3.9e-04^{\cdot}$	$1.5e-02$	$-4.9e-03$	$1.1e-02$	$6.6e-02^{**}$	$2.0e-01^{**}$
$(t-stat)$	$(4.2e+00)$	$(5.7e-01)$	$(4.8e-01)$	$(-3.1e+00)$	$(-1.0e+00)$	$(9.9e+00)$	$(-2.7e-01)$	$(1.7e+00)$	$(-1.3e+00)$	$(9.1e-01)$	$(7.3e+00)$	$(2.2e+01)$

to be continued on the next page

Table 3.4: Coefficient estimates of the VAR(3) model in (3.4.5). The endogenous state vector y_t consists of order book and order flow variables, see (3.4.6). T-statistics are reported in round brackes. "**", "*" and "·" denote significance at the 0.1%, 1% and 10% level respectively.

Variable	Order Book Variables						Order Flow Variables					
		Imbalances					Submissions		Cancellations		Executions	
$lag = 2\text{min}$	$spread$	I^D	I^H	$(D+H)$	$returns$	$vola$	$subbuy$	$subsel$	$canbuy$	$cansel$	$exebuy$	$exesel$
$spread_{t-2}$	$1.6e-01^{*}$	$-9.6e-02$	$1.5e-01$	$7.1e-04$	$-2.0e+01$	$-1.2e-02$	$7.7e-03$	$1.3e-03$	$8.4e-03$	$1.3e-04$	$1.1e-02$	$8.3e-03$
$(t{-}stat)$	$(1.7e+01)$	$(-3.0e-01)$	$(-4.8e-02)$	$(1.7e-01)$	$(-6.4e-01)$	$(-3.3e+00)$	$(7.4e-01)$	$(3.3e-01)$	$(7.7e-01)$	$(2.0e-01)$	$(8.3e-01)$	$(4.6e-01)$
I^D_{t-2}	$-1.7e-04$	$1.1e-01^{**}$	$-7.9e-03$	$2.7e-05$	$-2.8e-01$	$6.8e-05$	$-3.1e-05$	$8.0e-05$	$-1.1e-04$	$8.9e-05$	$4.1e-04$	$-2.8e-05$
$(t{-}stat)$	$(-2.0e-01)$	$(1.3e+01)$	$(2.1e-01)$	$(-1.6e-01)$	$(-1.8e-01)$	$(6.2e-02)$	$(-5.9e-02)$	$(1.4e-01)$	$(-3.3e-03)$	$(1.2e-01)$	$(4.0e-01)$	$(-6.4e-02)$
I^H_{t-2}	$-1.6e-04$	$9.9e-04$	$1.3e-01^{**}$	$-1.1e-04$	$-2.3e-02$	$-1.4e-04$	$1.8e-04$	$-1.2e-04$	$1.6e-04$	$-6.6e-05$	$-2.3e-04$	$1.1e-04$
$(t{-}stat)$	$(-1.2e-01)$	$(1.5e-01)$	$(1.6e+01)$	$(-4.4e-01)$	$(-1.2e-01)$	$(-2.6e-01)$	$(3.4e-01)$	$(-2.4e-02)$	$(4.3e-01)$	$(2.5e-02)$	$(-4.7e-01)$	$(-1.8e-01)$
$(D+H)_{t-2}$	$-6.4e-03$	$1.6e-01$	$-1.0e+00$	$2.1e-01^{**}$	$-2.0e+01$	$-1.3e-02$	$2.4e-02$	$3.5e-02$	$3.5e-02$	$3.4e-02$	$3.6e-02$	$3.3e-02$
$(t{-}stat)$	$(-2.2e-01)$	$(5.8e-01)$	$(-1.4e-01)$	$(2.4e+01)$	$(-4.8e-01)$	$(-1.0e+00)$	$(7.0e-01)$	$(9.9e-01)$	$(1.1e+00)$	$(9.5e-01)$	$(9.2e-01)$	$(8.1e-01)$
$returns_{t-2}$	$-3.4e-05$	$-2.3e-04$	$-3.9e-03$	$-4.0e-06$	$2.9e-02$	$-5.9e-06$	$5.1e-05$	$-7.5e-06$	$4.4e-05$	$1.6e-05$	$-1.9e-05$	$4.6e-05$
$(t{-}stat)$	$(-9.8e-01)$	$(-7.5e-01)$	$(-7.8e-01)$	$(-4.6e-01)$	$(3.5e+00)$	$(-1.1e+00)$	$(1.3e+00)$	$(-8.0e-01)$	$(1.1e+00)$	$(7.9e-01)$	$(-3.2e-01)$	$(9.7e-01)$
$vola_{\,t-2}$	$1.3e-02$	$1.6e-01$	$-8.1e-01$	$1.1e-02$	$-1.1e+01$	$3.1e-03$	$3.2e-02$	$9.3e-03$	$3.6e-02$	$-1.4e-03$	$2.7e-02$	$3.2e-02$
$(t{-}stat)$	$(4.8e-01)$	$(2.4e-01)$	$(-1.6e-01)$	$(1.3e+00)$	$(5.0e-02)$	$(2.9e-01)$	$(1.1e+00)$	$(6.8e-01)$	$(1.3e+00)$	$(3.3e-01)$	$(4.8e-01)$	$(6.1e-01)$
$subbuy_{t-2}$	$7.3e-03$	$-1.8e-01$	$3.6e+00$	$-3.2e-02^{*}$	$-3.0e+01$	$-2.1e-04$	$1.1e-01^{\cdot}$	$4.7e-02$	$2.8e-02$	$-8.5e-03$	$1.3e-01$	$6.0e-03$
$(t{-}stat)$	$(1.8e-01)$	$(-8.5e-02)$	$(4.1e-01)$	$(-2.8e+00)$	$(2.6e-02)$	$(1.6e-02)$	$(2.9e+00)$	$(8.7e-01)$	$(4.6e-01)$	$(-8.0e-01)$	$(2.0e+00)$	$(-9.9e-02)$
$subsel_{t-2}$	$4.1e-03$	$-1.1e-01$	$-3.3e-01$	$-1.9e-02$	$-1.3e+02^{\cdot}$	$1.2e-02$	$1.1e-02$	$1.8e-01^{**}$	$-1.1e-02$	$1.1e-02$	$4.0e-02$	$9.7e-02$
$(t{-}stat)$	$(6.1e-01)$	$(-1.9e-01)$	$(-3.6e-01)$	$(-1.8e+00)$	$(-4.8e+00)$	$(8.2e-01)$	$(-2.2e-01)$	$(7.6e+00)$	$(-1.0e+00)$	$(-2.5e-01)$	$(6.9e-01)$	$(1.5e+00)$
$canbuy_{t-2}$	$-1.8e-02$	$3.6e-01$	$-2.6e+00$	$3.3e-02^{\cdot}$	$5.3e+01$	$-4.0e-03$	$2.5e-02$	$-8.3e-03$	$1.1e-01^{\cdot}$	$4.8e-02$	$-1.1e-01$	$9.2e-03$
$(t{-}stat)$	$(-4.6e-01)$	$(1.7e-01)$	$(-2.0e-01)$	$(2.9e+00)$	$(1.6e-02)$	$(-3.0e-01)$	$(1.2e+00)$	$(3.4e-02)$	$(3.8e+00)$	$(1.8e+00)$	$(-1.8e+00)$	$(3.1e-01)$
$cansel_{t-2}$	$8.7e-04$	$-1.1e-01$	$-1.9e-01$	$2.1e-02$	$9.5e+01^{*}$	$-1.5e-02$	$1.3e-02$	$-6.3e-02^{*}$	$3.6e-02$	$1.0e-01$	$-1.8e-02$	$-7.8e-02$
$(t{-}stat)$	$(-2.1e-01)$	$(-4.2e-02)$	$(1.8e-01)$	$(2.1e+00)$	$(4.0e+00)$	$(-1.2e+00)$	$(8.4e-01)$	$(-2.0e+00)$	$(1.6e+00)$	$(5.5e+00)$	$(-1.3e-01)$	$(-1.1e+00)$
$exebuy_{t-2}$	$8.6e-03$	$-1.5e-02$	$-8.3e-01$	$9.8e-04$	$-6.3e+00$	$1.8e-03$	$7.1e-03$	$1.0e-02$	$1.3e-03$	$1.3e-02$	$1.1e-01^{**}$	$4.5e-02^{\cdot}$
$(t{-}stat)$	$(7.2e-01)$	$(-3.1e-01)$	$(-4.2e-02)$	$(1.2e-01)$	$(-2.1e-01)$	$(7.0e-01)$	$(7.9e-01)$	$(8.9e-01)$	$(-1.2e-01)$	$(1.4e+00)$	$(1.1e+01)$	$(4.5e+00)$
$exesel_{t-2}$	$-5.4e-03$	$3.7e-02$	$9.0e-02$	$1.1e-03$	$6.3e+00^{\cdot}$	$2.2e-04$	$6.2e-03$	$1.9e-03$	$5.4e-03$	$9.3e-04$	$3.9e-02^{**}$	$1.1e-01^{**}$
$(t{-}stat)$	$(-8.1e-02)$	$(5.8e-01)$	$(4.9e-01)$	$(4.3e-01)$	$(1.8e+00)$	$(2.6e-01)$	$(9.8e-01)$	$(4.0e-01)$	$(8.7e-01)$	$(2.9e-01)$	$(4.1e+00)$	$(1.2e+01)$

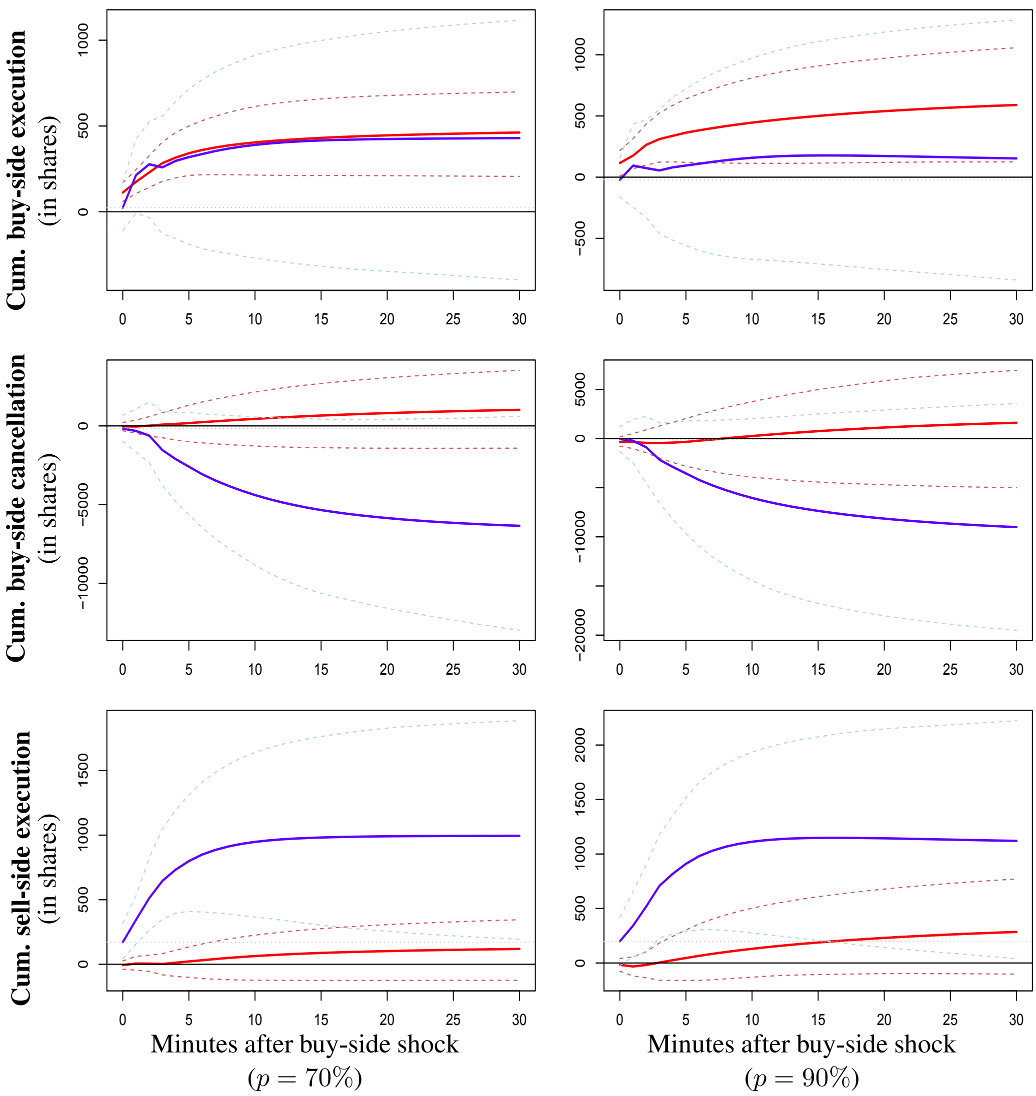
to be continued on the next page

Table 3.4: Coefficient estimates of the VAR(3) model in (3.4.5). The endogenous state vector y_t consists of order book and order flow variables, see (3.4.6). T-statistics are reported in round brackes. "**", "*" and "·" denote significance at the 0.1%, 1% and 10% level respectively.

	Order Book Variables						Order Flow Variables					
Variable		Imbalances					Submissions		Cancellations		Executions	
$lag = 3\text{min}$	$spread$	I^D	I^H	$(D+H)$	$returns$	$vola$	$subbuy$	$subsel$	$canbuy$	$cansel$	$exebuy$	$exesel$
$spread_{t-3}$	$1.5e-01^{*}$	$1.9e-02$	$1.9e-01$	$1.6e-03$	$7.6e+00$	$-6.9e-03$	$1.0e-02$	$9.0e-03$	$1.1e-02$	$1.3e-02$	$1.6e-02$	$1.9e-02$
$(t{-}stat)$	$(1.8e{+}01)$	$(5.1e{-}02)$	$(4.3e{-}01)$	$(7.3e{-}01)$	$(-1.1e{-}01)$	$(-2.3e{+}00)$	$(1.2e{+}00)$	$(9.5e{-}01)$	$(1.2e{+}00)$	$(1.4e{+}00)$	$(1.0e{+}00)$	$(1.4e{+}00)$
I^D_{t-3}	$3.8e-04$	$8.6e-02^{**}$	$-1.2e-02$	$-1.0e-04$	$3.6e-01$	$1.7e-05$	$-3.4e-04$	$-3.4e-04$	$-4.5e-05$	$-3.5e-04$	$-1.9e-04$	$-8.9e-04$
$(t{-}stat)$	$(3.6e{-}01)$	$(1.0e{+}01)$	$(-4.2e{-}01)$	$(-5.2e{-}01)$	$(7.7e{-}02)$	$(-7.7e{-}02)$	$(-4.0e{-}01)$	$(-5.9e{-}01)$	$(-3.2e{-}02)$	$(-5.4e{-}01)$	$(-5.0e{-}01)$	$(-6.3e{-}01)$
I^H_{t-3}	$7.3e-05$	$4.1e-03$	$1.1e-01^{**}$	$2.9e-05$	$-3.9e-01$	$-6.3e-05$	$6.8e-05$	$-4.9e-05$	$5.4e-05$	$-2.9e-05$	$-2.4e-04$	$-1.3e-05$
$(t{-}stat)$	$(2.9e{-}01)$	$(1.3e{-}02)$	$(1.3e{+}01)$	$(-1.5e{-}01)$	$(1.8e{-}02)$	$(-4.9e{-}01)$	$(-2.6e{-}02)$	$(2.8e{-}01)$	$(2.2e{-}02)$	$(2.3e{-}01)$	$(-7.0e{-}02)$	$(1.1e{-}02)$
$(D+H)_{t-3}$	$-4.9e-02$	$3.2e-01$	$-4.3e-01$	$1.9e-01^{**}$	$-2.4e+01$	$-3.6e-03$	$-2.7e-03$	$1.4e-02$	$8.6e-03$	$2.0e-02$	$-1.2e-02$	$-3.7e-02$
$(t{-}stat)$	$(-1.1e{+}00)$	$(5.7e{-}01)$	$(6.1e{-}01)$	$(2.3e{+}01)$	$(-3.1e{-}01)$	$(-4.0e{-}01)$	$(1.1e{-}02)$	$(6.0e{-}01)$	$(4.3e{-}01)$	$(7.5e{-}01)$	$(-1.6e{-}01)$	$(-7.4e{-}01)$
$returns_{t-3}$	$-5.3e-06$	$-1.1e-04$	$-2.2e-03$	$-1.4e-06$	$-8.2e-03$	$6.5e-06$	$2.0e-05$	$8.6e-07$	$1.6e-05$	$5.1e-06$	$1.1e-05$	$1.1e-05$
$(t{-}stat)$	$(-5.0e{-}02)$	$(-1.1e{-}01)$	$(-5.4e{-}01)$	$(5.0e{-}02)$	$(-1.3e{+}00)$	$(2.8e{-}01)$	$(3.3e{-}01)$	$(-7.0e{-}01)$	$(-2.1e{-}01)$	$(-1.7e{-}01)$	$(2.0e{-}02)$	$(-2.0e{-}01)$
$vola_{\,t-3}$	$4.1e-02$	$-2.7e-01$	$8.0e-01$	$1.8e-03$	$-9.8e+00$	$-3.2e-02^{\cdot}$	$5.7e-02^{\cdot}$	$7.6e-02^{*}$	$5.6e-02^{\cdot}$	$8.7e-02^{**}$	$1.1e-01^{*}$	$1.2e-01^{**}$
$(t{-}stat)$	$(1.7e{+}00)$	$(-5.2e{-}01)$	$(1.9e{-}01)$	$(2.5e{-}01)$	$(-3.7e{-}01)$	$(-3.9e{+}00)$	$(3.0e{+}00)$	$(3.5e{+}00)$	$(2.9e{+}00)$	$(4.2e{+}00)$	$(3.5e{+}00)$	$(4.0e{+}00)$
$subbuy_{t-3}$	$7.4e-03$	$-1.4e-01$	$4.3e+00$	$-2.4e-02^{\cdot}$	$1.3e+02$	$9.3e-03$	$5.5e-02$	$1.5e-02$	$-1.9e-02$	$-4.2e-02$	$3.4e-02$	$-1.9e-02$
$(t{-}stat)$	$(-2.0e{-}01)$	$(1.5e{-}01)$	$(9.3e{-}01)$	$(-2.3e{+}00)$	$(2.4e{+}00)$	$(6.8e{-}01)$	$(1.7e{+}00)$	$(2.2e{-}01)$	$(-2.9e{-}01)$	$(-1.7e{+}00)$	$(4.4e{-}01)$	$(-2.3e{-}01)$
$subsel_{t-3}$	$2.9e-02$	$-5.8e-02$	$5.1e-01$	$-1.6e-02$	$-1.0e+02^{\cdot}$	$3.8e-03$	$1.3e-02$	$1.7e-01^{*}$	$3.6e-03$	$-5.7e-03$	$-1.6e-02$	$1.5e-02$
$(t{-}stat)$	$(7.6e{-}01)$	$(-2.0e{-}01)$	$(-1.5e{-}01)$	$(-1.6e{+}00)$	$(-3.7e{+}00)$	$(7.9e{-}01)$	$(2.8e{-}02)$	$(7.8e{+}00)$	$(-4.3e{-}01)$	$(-5.7e{-}01)$	$(-9.2e{-}02)$	$(3.0e{-}01)$
$canbuy_{t-3}$	$-1.7e-02$	$1.8e-01$	$-3.6e+00$	$2.3e-02^{\cdot}$	$-1.2e+02$	$-2.2e-03$	$3.6e-02$	$1.3e-02$	$1.1e-01^{*}$	$7.0e-02$	$-3.3e-02$	$1.6e-02$
$(t{-}stat)$	$(-1.6e{-}01)$	$(-3.1e{-}01)$	$(-8.5e{-}01)$	$(2.3e{+}00)$	$(-2.3e{+}00)$	$(-2.1e{-}01)$	$(1.4e{+}00)$	$(3.7e{-}01)$	$(3.5e{+}00)$	$(2.4e{+}00)$	$(-3.8e{-}01)$	$(2.7e{-}01)$
$cansel_{t-3}$	$-2.5e-02$	$-7.9e-02$	$-1.2e+00$	$2.0e-02$	$9.4e+01$	$-1.6e-03$	$4.8e-02$	$-3.9e-02$	$5.7e-02^{\cdot}$	$1.3e-01$	$5.7e-02$	$4.2e-02$
$(t{-}stat)$	$(-6.2e{-}01)$	$(1.8e{-}02)$	$(2.1e{-}01)$	$(2.2e{+}00)$	$(3.2e{+}00)$	$(-5.2e{-}01)$	$(2.4e{+}00)$	$(-1.4e{+}00)$	$(2.8e{+}00)$	$(6.4e{+}00)$	$(1.3e{+}00)$	$(1.2e{+}00)$
$exebuy_{t-3}$	$-1.6e-03$	$-5.7e-03$	$5.2e-01$	$-2.5e-04$	$-2.7e+00$	$4.0e-03$	$-4.5e-04$	$-5.4e-03$	$-6.4e-03$	$-1.1e-03$	$9.4e-02^{**}$	$2.9e-02$
$(t{-}stat)$	$(-9.2e{-}02)$	$(1.7e{-}01)$	$(2.0e{-}01)$	$(-5.0e{-}01)$	$(-1.2e{+}00)$	$(1.4e{+}00)$	$(-6.0e{-}02)$	$(-5.3e{-}01)$	$(-1.1e{+}00)$	$(1.5e{-}01)$	$(9.7e{+}00)$	$(3.0e{+}00)$
$exesel_{t-3}$	$5.5e-03$	$1.4e-02$	$-5.6e-01$	$-3.6e-05$	$8.2e+00$	$7.5e-04$	$8.2e-03$	$9.6e-04$	$7.7e-03$	$1.0e-03$	$4.2e-02^{**}$	$1.0e-01^{**}$
$(t{-}stat)$	$(6.1e{-}01)$	$(6.2e{-}04)$	$(-6.1e{-}01)$	$(-2.2e{-}01)$	$(9.1e{-}01)$	$(2.3e{-}01)$	$(1.1e{+}00)$	$(1.4e{-}01)$	$(1.1e{+}00)$	$(2.3e{-}01)$	$(4.5e{+}00)$	$(1.1e{+}01)$
$const$	$1.4e-01$	$-8.0e-02$	$-2.7e+00$	$3.0e-01^{**}$	$6.0e+01$	$3.5e-02^{\cdot}$	$1.3e-01^{*}$	$8.7e-02$	$1.1e-01^{*}$	$8.8e-02$	$8.7e-02$	$6.2e-02$
$(t{-}stat)$	$(4.1e{+}00)$	$(6.6e{-}01)$	$(-4.5e{-}01)$	$(3.1e{+}01)$	$(9.0e{-}01)$	$(2.9e{+}00)$	$(3.9e{+}00)$	$(2.7e{+}00)$	$(3.3e{+}00)$	$(2.8e{+}00)$	$(1.4e{+}00)$	$(1.1e{+}00)$

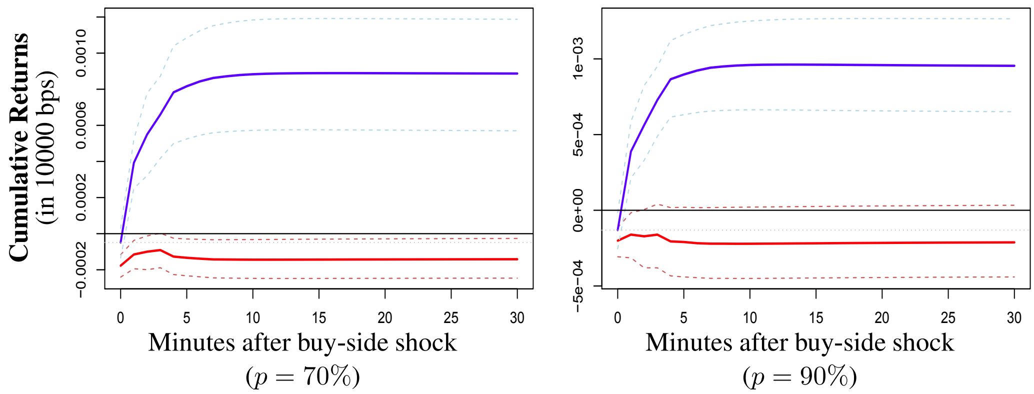
Appendix 3.D Impulse Response Estimates: Order Flows

Figure 3.6: Cross-sectional averages of the cumulative impulse response of order flows due to a buy-side (i.e. positive) shock in hidden (blue) and displayed (red) order imbalances. In this case the plain hidden I_H and displayed I_D imbalances of the endogenous state vector (3.4.6) are replaced by their corresponding dummies. The dummy variables are defined with value one exceeding the $p-$ quantile level of total imbalances I_T and zero otherwise. 95% bootstrap confidence intervals shown in corresponding light colors and dashed lines.



Appendix 3.E Impulse Response Estimates: Returns

Figure 3.7: Cross-sectional averages of the cumulative impulse response of returns due to a buy-side (i.e. positive) shock in hidden (blue) and displayed (red) order imbalances. In this case the plain hidden I_H and displayed I_D imbalances of the endogenous state vector (3.4.6) are replaced by their corresponding dummies. The dummy variables are defined with value one exceeding the $p-$ quantile level of total imbalances I_T and zero otherwise 95% bootstrap confidence intervals shown in corresponding light colors and dashed lines.



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Selbständigkeitserklärung

Ich bezeuge durch meine Unterschrift, dass meine Angaben über die bei der Abfassung meiner Dissertation benutzten Hilfsmittel, über die mir zuteil gewordene Hilfe sowie über frühere Begutachtungen meiner Dissertation in jeder Hinsicht der Wahrheit entsprechen.

Berlin, 24. Juli 2013

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